E&M Problem Set 12

Due ***Tuesday***, April 29 at 4pm

Reading Note: For this last assignment, I suggest reading Sections 7.2 - 7.3.3 (for inductance and Maxwell's equations), 8.1 (For a discussion of the Poynting Vector), and 9.1 - 9.2 (on waves and electromagnetic waves).

1. **Griffiths Problem 7.22:** Find the self-inductance per unit length of a long solenoid, of radius R, carrying n turns per unit length.

2. **Griffiths Problem 7.26 tweaked:** Find the energy stored in the magnetic field in a section of length ℓ of a long solenoid (radius R, current I, n turns per unit length),

   (a) using Equation 7.29, $W = \frac{1}{2}LI^2$ (you found L in Problem 7.22);

   (b) using Equation 7.34, $W = \frac{1}{2\mu_0} \int\limits_{\text{all space}} B^2 d\tau$;

   (c) Compare your results from (a) and (b), do they make sense?

3. **Griffiths Problem 7.31 tweaked:** A fat wire, radius a, carries a constant current I, uniformly distributed over the cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as shown in Figure 7.43 (below).

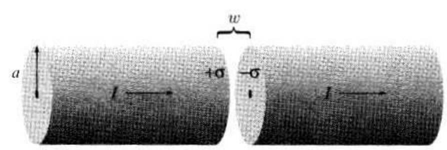

Figure 7.43

   (a) There is no current in the gap itself. While ignoring "fringing magnetic fields" of the wire segements (there are none of any significance in the gap), argue that there must be a magnetic field in the gap? **HINT:** The ends of the wire are behaving like a charging capacitor. What's happening to the charge on the "plates" of this capacitor? What does that mean in terms of the $\vec{E}$ and $\vec{B}$ fields?

   (b) Find the magnetic field in the gap, at a distance $s < a$ from the axis.

4. **Griffiths Problem 8.02:** Consider the charging capacitor in Problem 7.31.

   (a) Find the electric and magnetic fields in the gap, as functions of the distance s from the axis and the time t. (Assume the charge is zero at $t = 0$.) **HINT:** You should have determined the magnetic field in doing Problem 7.31. Depending on how you attacked that problem, you may well have determined the electric field in the gap in terms of current and time, if so, use that result here. Otherwise, derive it from what you found in Problem 7.31.

   (b) Find the energy density u_{em} and the Poynting vector $\vec{S}$ in the gap. Note especially the *direction* of $\vec{S}$. Check that equation 8.14 is satisfied.

   (c) Determine the total energy in the gap, as a function of time. Calculate the total power flowing into the gap, by integrating the Poynting vector over the appropriate surface. Check that the power input is equal to the rate of increase of energy in the gap (equation 8.9 - in this case $W = 0$, because there is no charge in the gap) [If you're worried about the fringing fields, do it for a volume of radius $b < a$ well inside the gap.]

5. **Griffiths Problem 9.09:** Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and phase angle zero that is

   (a) traveling in the negative x direction and polarized in the z direction;

   (b) traveling in the direction from the origin to the point (1,1,1), with polarization parallel to the xz plane.

   In each case, sketch the wave, and give the explicit cartesian components of the propogation vector $\vec{k}$ and polarization $\hat{n}$.

6. **Griffiths Problem 9.10 tweaked:** The intensity of sunlight hitting the earth is about $1300 W/m^2$ (at the top of the atmosphere).

   (a) If the atmosphere was a perfect absorber, what pressure would the sunlight exert?

   (b) How about if the Earth's atmosphere was a perfect reflector?

   (c) Let's assume the Earth's atmophere is indeed perfectly reflective (not true, since you can see the Sun during the daytime). If "an atmosphere" of pressure is $1.03 \times 10^5 Pa$, what fraction of atmospheric pressure does the pressure of sunlight amount to?