E&M Problem Set 10

Due *Wednesday, April 9 at 4pm*

NOTE: If you can generate a PDF of your problem set solutions, please email it to me at *juan@cabanela.com*. If you can not do this yourself, provide a **single-sided copy** of your problem set solutions to Joy by 3pm on Wednesday.

1. **Griffiths Problem 6.01 tweaked:** Calculate the torque exerted on the square loop shown in Fig. 6.6 (below), due to the circular loop (assume r is much larger that a or b). If the square loop is free to rotate, what will its equilibrium orientation be?

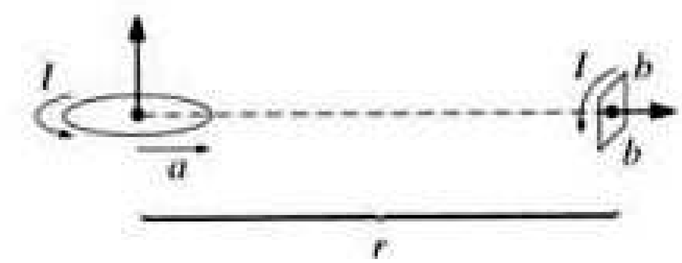

Figure 6.06

Clearly state, in English, which direction the square loop will turn in response to this torque. **HINT:** There are several forms of the equation for the magnetic field of a dipole given in the book. I would suggest using the coordinate-free version in equation 5.87 in this problem to reduce the complexity of the math.

2. **Griffiths Problem 6.07:** An infinitely long circular cylinder carries a uniform magnetization $\vec{M}$ parallel to its axis. Find the magnetic field (due to $\vec{M}$) inside and outside the cylinder.

3. **Griffiths Problem 6.09 tweaked:** A short circular cylinder of radius a and length L carries a "frozen-in" uniform magnetization $\vec{M}$ parallel to its axis. Find the bound current, and sketch the magnetic field of the cyliner. (Make three sketches: one for $L \gg a$, one for $L \ll a$, and for $L \approx a$.) **HINT:** Very few explicit calculations are involved here. This is mostly a conceptual problem.

4. **Griffiths Problem 6.10:** An iron rod of length L and square cross section (side a), is given a uniform longitudinal magnetization $\vec{M}$, and then bent around into a circle with a narrow gap (width w), as shown in Fig. 6.14 (below). Find the magnetic field at the center of the gap, assuming $w \gg a \gg L$. [*Hint:* Treat it as the superposition of a *complete* torus plus a square loop with reversed current. You determined the magnetic field in a square current loop in an earlier problem set.]

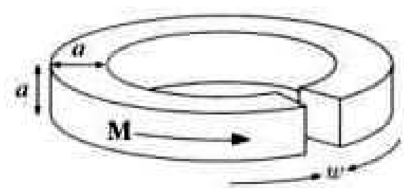

Figure 6.14

5. **Griffiths Problem 6.12:** An infinitely long cylinder, of radius R, carries a "frozen-in" magnetization, parallel to the axis,

$$\vec{M} = ks\hat{z}$$

where k is a constant and s is the distance from the axis; there is no free current anywhere. Find the magnetic field inside and outside the cylinder by two different methods:

   (a) As in Section 6.2, locate all the bound currents, and calculate the field they produce.

   (b) Use Ampères law (in the form of Eq.6.20) to find $\vec{H}$, and then get $\vec{B}$ from Equation 6.18, $\vec{H} \equiv \frac{1}{\mu_0}\vec{B} - \vec{M}$. (Notice that the second method is much faster, and avoids any explicit reference to the bound currents.)

6. **Griffiths Problem 6.25 tweaked:** A familiar toy consists of donut-shaped permanent magnets (magnetization parallel to the axis), which slide frictionlessly on a vertical rod (see Fig. 6.31, below). Treat the magnets as dipoles, with mass m_d and dipole moment $\vec{m}$.

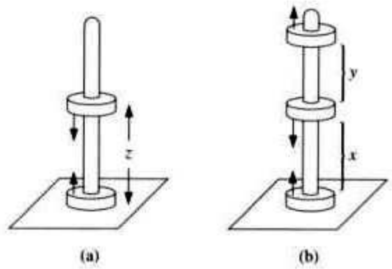

Figure 6.31

(a) If you put two back-to back magnets on the rod, the upper one will "float" — the magnetic force upward balancing the gravitational force downward (See Figure 6.31a above). If you treat each magnets as a dipole, their (individual) magentic field is given by equation 5.86. Explain *why* the magnets exert a force on each other? **HINT:** Under what conditions will a magnetic dipole feel a force as opposed to just a torque?

(b) At what height (z) does it float? [*Answer*: $[3\mu_0 m^2/2\pi m_d g]^{1/4}$]

(c) If you now add a *third* magnet parallel to the bottom one (See Figure 6.31b above), what is the *ratio* of the two heights, x/y? (Determine the actual number, to three significant digits.) [*Answer*: 0.8501] **HINT:** Determine expressions for all the forces acting on the top and middle magnets. Both magnets are in equilibrium, so you will be able to set up two equations that must both be true in equilibrium. Combine them to come up with an expression containing terms with x/y in them and some constants and nothing else. At this stage, solve for x/y numerically using *Maple* or *Mathematica*. In *Maple*, you can solve an expression numerically using the `solve` command. For example, to solve for when $x^2 - 3x + 0.01 = 0$, you type

```
solve(x^2-3*x+0.01=0, x);
```

and both roots of the quadratic will be computed. If there are multiple roots (and there will be in this case), *Maple* resorts to telling you what the solution is the `RootOf` instead of numerically solving for it. You can force a numerical solution using evalf ("evaluate as float") command such that:

```
evalf(solve(x^2-3*x+0.01=0, x));
```

If you use *Maple* or *Mathematica* to solve this problem, you must include the output or state the commands you used explicitly.