E&M Problem Set 9

Due ***Tuesday***, March 25 at 4pm

NOTE: As an experiment this week, I am asking everyone who is able to scan their homework to try sending me your solutions to this problem set via email in PDF format (or GIF if you can't handle PDF). I've got a solution worked out for the rest of your regarding the next problem set which will be due while I am in Chile, but I want to get a feel for how many of you can do this without help.

1. **Griffiths Problem 5.19 (tweaked):**

   (a) Find the density ρ of mobile charges in a piece of copper, assuming each atom contributes one free electron. **Useful Physical Constants:** The atomic mass of copper, $M = 64\frac{gm}{mol}$, the density of copper is $d = 9.0\frac{gm}{cm^3}$, and Avagadro's Number is $A = 6.0 \times 10^{23}/mol$.

   (b) Calculate the average electron velocity in copper wire 1mm in diameter, carrying a current of 1 A.

   (c) **(1 pt Extra Credit)** Your answer to part (b) should show the average electron velocity is literally a *snail's* pace. How, then, can you carry on a long distance telephone conversation?

   (d) Consider two such wires laying parallel to one another. Imagine for a moment all the stationary positive ions (which are providing charge neutrality) were removed. Compare the magnetic force per unit length to the electrical force per unit length between two such wires. In other words, write an expression showing how many times greater than the magnetic force per unit length is compared to the magnetic force per unit length, f_e/f_m. **HINT:** You worked out the magnetic force per unit length between two current carrying wires in example 5.9 of the textbook. The electric field of a line charge was worked out in Example 2.1 of the textbook. Work out an expression for the electric force per unit length in terms of currents and use the fact that the speed of light $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$ to allow you compare electric force per unit length to magnetic force per unit length.

2. **Griffiths Problem 5.46:** The magnetic field on the axis of a circular current loop, $B(z) = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}$ (equation 5.38). is far from uniform (it falls off sharply with increasing z). You can produce a more nearly uniform field by using two such loops a distance d apart (See Figure 5.60, below).

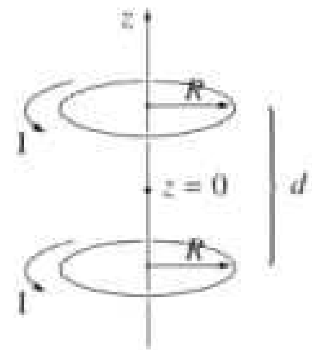

Figure 5.60

   (a) Find the field (B) as a function of z, and show that $\frac{\partial B}{\partial z}$ is zero at the point midway between them ($z = 0$). Now, if you pick d just right the *second* derivative of B will also vanish at the midpoint. This arrangement is known as the **Helmholtz coil**; it's a convenient way of producing relatively uniform fields in the laboratory.

   (b) Determine d such that $\frac{\partial^2 B}{\partial z^2} = 0$ at the midpoint, and find the resulting magnetic field at the center. $[Answer: \frac{8\mu_0 I}{5\sqrt{5}R}]$

3. **Griffiths Problem 5.14:** A thick slab extending from $z = -a$ to $z = +a$ carries a uniform volume current $\vec{J} = J\hat{x}$ (see Figure 5.41, below). Find the magnetic field, as a function of z, both inside and outside the slab.

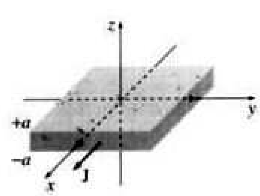

Figure 5.41

4. **Griffiths Problem 5.20 (modified):** Recall the general rule (expressed in equation 1.46) that the divergence of a curl is always zero.

   (a) Using the differential form of Ampére's law, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ and the continuity equation $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$ to write an expression for $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B})$.

   (b) At first blush, the expression you computed in part (a) doesn't equal zero... Show that differential form of Ampére's law you used in part (a) can not be valid except in the realm of *magnetostatics* (**HINT:** Read section 5.2.1 for a discussion of the conditions necessary for a steady current as necessary in magnetostatics.

   (c) Verify that there is no other such problems taking the curl of the three other Maxwell equations (as listed on page 232 of your textbook).

5. **Griffiths Problem 5.29:** Use the results of Example 5.11 to find the $\vec{B}$ field inside a uniformly charged sphere, of total charge Q and radius R, which is rotating at a constant angular velocity ω.

6. **Griffiths Problem 5.34:** A circular loop of wire, with radius r, lies in the xy plane, centered at the origin, and carries a current I running counterclockwise as viewed from the positive z axis.

   (a) What is its magnetic dipole moment?

   (b) What is the approximate magnetic field at points far from the origin?

   (c) Show that, for points on the z axis, your answer is consistent with the exact field $B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$ (equation 5.38) from Example 5.6, when $z \gg R$.

7. **Griffiths Problem 5.56:** A thin uniform donut, carrying charge Q and mass M, rotates about its axis as shown in Figure 5.64 (below).

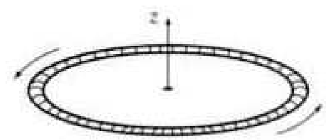

Figure 5.64

   (a) Find the ratio of its magnetic dipole moment to its angular momentum. This is called the **gyromagnetic ratio** (or **magnetomechanical ratio**).

   (b) What is the gyromagnetic ratio for a uniform spinning sphere? [This requires no new calculation; simply decompose the sphere into infinitesimal rings, and apply the result of part (a).]

   (c) According to quantum mechanics, tha angular momentum of a spinning electron is $\frac{\hbar}{2}$, where $\hbar$ is Planck's constant. What, then, is the electron's magnetic dipole moment, in $A{\cdot}m^2$? (This semiclassical value is actually off by a factor of almost exactly 2. Dirac's relativisitic electron theory got the 2 right, and Feynman, Schwinger, and Tomonoga later calculated tiny further corrections. The determination of the electron's magnetic dipole moment remains the finest achievement of quantum electrodynamics, and exhibits perhaps the most stunningly precise agreement between theory and experiment in all of physics. Incidentally, the quantity $(e\hbar/2m)$, where e is the charge of the electron and m is its mass, is call the **Bohr magneton**.)