SPECIAL NOTE: I could not find a symbol that exactly matches the book's script r for the separation vector. Instead I am using the following notation: $\vec{\mathfrak{r}} = \vec{r} - \vec{r}'$, $\mathfrak{r} = |\vec{\mathfrak{r}}|$ and $\hat{\mathfrak{r}} = \vec{\mathfrak{r}}/\mathfrak{r}$.

1. **Griffiths Problem 4.16 (tweaked):** Suppose the electric field inside a large piece of dielectric is $\vec{E}_0$, so that the electric displacement is $\vec{D}_0 = \epsilon_0 \vec{E}_0 + \vec{P}$. Imagine cavities are hollowed out of the material. Assume the cavities are small enough that $\vec{P}$, $\vec{E}_0$, and $\vec{D}_0$ are essentially uniform. For each of the following cavity shapes, find the electric field $\vec{E}$ at the center of the cavity in terms of $\vec{E}_0$ and $\vec{P}$ and the displacement $\vec{D}$ at the center of the cavity in terms of $\vec{D}_0$ and $\vec{P}$. **HINT:** Carving out a cavity is the same as superimposing an object of the same shape but opposite polarization.

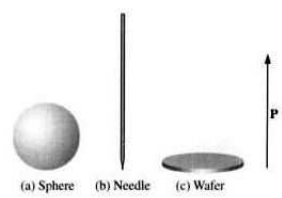


Figure 4.19

   (a) Find $\vec{E}$ and $\vec{D}$ at the center of a small spherical cavity (Figure 4.19a, above). **NOTE:** We derived the electric field of a uniformly polarized spherical dielectric in Example 4.2. This might come in handy.

   (b) Do the same for a long needle-shaped cavity (assume 'needle' here is the same as 'thin, long cylinder') running parallel to $\vec{P}$ (Figure 4.19b, above). **HINT:** For this problem, consider the location of bound charges in the spherical cavity and remember the cavity is *long*. Very little math is involved if you make this assumption.

   (c) Do the same for a thin wafer-shaped cavity perpendicular to $\vec{P}$ (Figure 4.19c, above). **HINT:** Consider where the bound charge accumilates. You must successfully determine the bound surface charge density at the upper/lower surface. Since this wafer is much thinner than it is wide, what you basically have here is a capacitor. The electric field within a capacitor was discussed in Example 2.5.

(a) As suggested by the initial hint, the electric field $\vec{E}$ at the center of the cavity should be the same as the initial electric field $\vec{E}_0$ plus the electric field at the center of a sphere with uniform polarization $-\vec{P}$ (or *minus* the electric field of a sphere with uniform polarization $\vec{P}$). Equation 4.14 summarizes the result of Example 4.2, that the electric field in a uniformly polarized sphere (aka UPS) is also uniform:

$$\vec{E}_{\text{UPS}} = -\frac{\vec{P}}{3\epsilon_0}. \tag{1}$$

Adding the electric field of a UPS with polarization $-\vec{P}$ to the initial electric field $\vec{E}_0$ we have the electric field at the center of the spherical cavity

$$\vec{E} = \vec{E}_0 + \frac{\vec{P}}{3\epsilon_0}. \tag{2}$$

As for the electric displacement $\vec{D}$, recall electric displacement is defined as (see equation 4.21):

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P} \tag{3}$$

but in the cavity, the polarization $\vec{P}$ must be zero (since polarization is only defined within a dielectric medium). Therefore the electric displacement at the center of the spherical cavity must be:

$$\vec{D} = \epsilon_0 \vec{E} = \epsilon_0 \vec{E}_0 + \frac{\vec{P}}{3} = (\vec{D}_0 - \vec{P}) + \frac{\vec{P}}{3} = \vec{D}_0 - \frac{2\vec{P}}{3} \tag{4}$$

(b) This is the same as the electric field $\vec{E}_0$ minus the electric field due to a thin uniformly polized needle with polarization $\vec{P}$ aligned with the axis of the needle. But the bound charges in a thin needle shaped polarized object accumilate at the ends. So near the center of the needle, they have a very small contribution to the electric field, so

$$\vec{E} = \vec{E}_0. \tag{5}$$

Therefore the electric displacement is

$$\vec{D} = \epsilon_0 \vec{E} = \epsilon_0 \vec{E}_0 = \vec{D}_0 - \vec{P}. \tag{6}$$

(c) To compute the electric field in this cavity, we need to add the electric field of a dielectric wafer with polarization $\vec{P}$. Following the hint, the bound charge on the upper and lower surfaces can be computed using equation 4.11

$$\sigma_b = \vec{P} \cdot \hat{n} \tag{7}$$

where $\hat{n}$ is parallel to $\vec{P}$ on the upper surface and anti-parallel on the lower surface. Therefore in this case

$$\sigma_b = \pm P \tag{8}$$

where the sign is positive on the upper surface and negative on the lower surface. Examining Example 2.5, it is clear the electric field in such a dielectric wafer must be $\vec{E}_{\text{wafer}} = -\frac{\sigma_b}{\epsilon_0} = -\frac{\vec{P}}{\epsilon_0}$, therefore the electric field at the center of the wafer cavity is:

$$\vec{E} = \vec{E}_0 - \vec{E}_{\text{wafer}} = \vec{E}_0 + \frac{\vec{P}}{\epsilon_0} \tag{9}$$

Given this electric field, the electric displacement at the center of the wafer is

$$\vec{D} = \epsilon_0 \vec{E} = \epsilon_0 \vec{E}_0 + \vec{P} = \vec{D}_0. \tag{10}$$

2. **Griffiths Problem 5.01 (tweaked):** A particle of charge q enters a region of uniform magnetic field $\vec{B}$ (pointing *into* the page). The field deflects the particle a distance d above the original line of flight, as shown in Figure 5.8 (shown below).

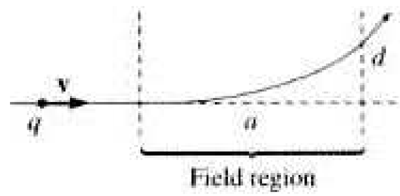
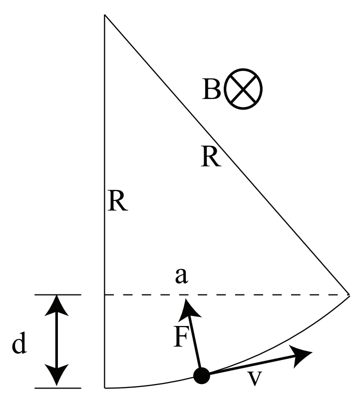


Figure 5.8

(a) If $\vec{B}$ is uniform, what is the shape of the path the charged particle follows while in the $\vec{B}$ field assuming no electric fields are present? Relate a and d to a property of that shape.

(b) Is the charge positive of negative? In terms of a, d, B, and q, find the momentum of the particle.

(a) A particle of charge q enters a region with magnetic field $\vec{B}$ and follows the trajectory shown below.



Since the force on a charged particle must be proportional to $\vec{v} \times \vec{B}$ via Lorentz' Law, it must act perpendicular to the charge's velocity, which leads to circular motion. Therefore the shape of the path is a circular arc with radius R indicated in the figure above. We can relate a and d to R using the triangle in the figure above and Pythagoras' Theorem,

$$R^2 = a^2 + (R-d)^2 \quad \Rightarrow \quad R = \frac{a^2+d^2}{2d}. \tag{11}$$

(b) As noted above, the force on the particle must pointperpendicular to the velocity, in this case in the direction shown in the figure above. Since $\vec{v} \times \vec{B}$ also points in that direction the particle's charge must be positive.
The magnitude of the magnetic force on the particle is $F = qvB$. Since this magnetic force is acting as a centripetal force here, the acceleration can be written v^2/R, where R is the radius of the circular arc on which the particle moves. We can solve for the momentum, mv, from Newton's second law,

$$qvB = mv^2/R \quad \Rightarrow \quad mv = qBR. \tag{12}$$

Using the relation between a and d and R from equation 11, we have a final expression for the momentum of the charged particle

$$p = mv = qB\left(\frac{a^2+d^2}{2d}\right) \tag{13}$$

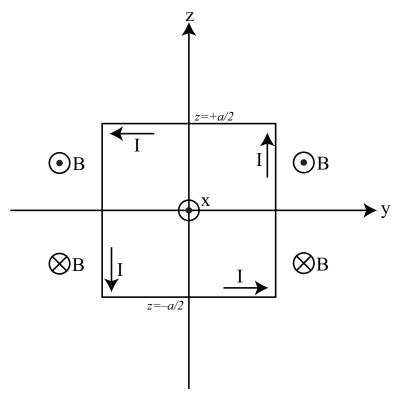
(Thanks to Dr. Craig for providing the initial draft of this solution in LaTeX form.)

3. **Griffiths Problem 5.04:** Suppose the magnetic field in some region has the form

$$\vec{B} = kz\hat{x} \tag{14}$$

(where k is a constant). Find the force on a square loop (side a), lying in the yz plane and centered at the origin, if it carries a current I, flowing counterclockwise, when you look down the x axis. **HINT:** Start by drawing the situation described here, showing your view looking down the x-axis (that is, so the x-axis is pointing right at your eye).

The square loop of wire shown below is in a magnetic field $\vec{B} = kz\hat{x}$. In the diagram below the current flows counterclockwise, the x axis is coming out of the page, and the direction of the magnetic field for $z > 0$ and for $z < 0$ is indicated.



The magnetic force on a segment of wire of length $d\vec{\ell}$ is

$$d\vec{F} = I d\vec{\ell} \times \vec{B}. \tag{15}$$

On the right and left hand sides of the square wire loop, the force is zero because for each segment of the wire above the z-axis, where $d\vec{\ell} \times \vec{B} = -\hat{y}$ there is another segment below the z-axis where $d\vec{\ell} \times \vec{B} = +\hat{y}$ so the total force cancels. The force on top side of the square wire loop will be the same as the force on the bottom side because the magnitude of the magnetic field is the same and the right-hand rule gives the same direction, $\hat{z}$, for the force. Along segment 1 the magnetic field is constant, $B = k(a/2)$, so the total force on that segment is just

$$F_1 = IaB = \frac{ka^2 I}{2}, \tag{16}$$

and the total force overall is $F = 2F_1 = ka^2 I$. *(Thanks to Dr. Craig for providing the initial draft of this solution in LaTeX form.)*

4. **Griffiths Problem 5.08:**

   (a) Find the magnetic field at the center of a square loop, which carries a steady current I. Let R be the distance from center to side (Figure 5.22 [below]).

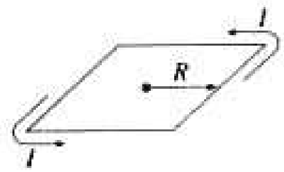


Figure 5.22

   (b) Find the field at the center of a regular n-sided polygon, carrying a steady current I. Again, let R be the distance from the center to any side.

   (c) Check that your formula reduces to the field at the center of a circular loop, in the limit $n \to \infty$.

HINT: Using the general results of Example 5.5 for the magnetic field due to a *segment* of straight uniform current will make this a very easily managable problem.

Starting with equation (5.35) from the textbook, we know the magnetic field at any given location due to a segment of straight uniform current is:

$$B = \frac{\mu_0 I}{4\pi s}\left(\sin\theta_2 - \sin\theta_1\right). \tag{17}$$

we can solve for the $\vec{B}$ field in each of these cases:

(a) At the center of a square loop of current, each side can be considered a straight line current extending from $\theta_1 = \frac{-\pi}{4}$ to $\theta_2 = \frac{\pi}{4}$ with $s = R$. So using equation 17 the $\vec{B}$ field due to each side is:

$$B = \frac{\mu_0 I}{4\pi R}\left(\sin\left(\frac{\pi}{4}\right) - \sin\left(-\frac{\pi}{4}\right)\right) = \frac{\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{4}\right) = \frac{\mu_0 I}{2\sqrt{2}\pi R}. \tag{18}$$

Via the right-hand rule, we know the magnetic field points “up” at the center of the square (as shown in Figure 5.22), or if I consider the loop to lie in the xy plane, $\vec{B}$ points in the $\hat{z}$ direction. Add up the contributions of the 4 sides of the sqaure and we have the total $\vec{B}$ at the center of the loop:

$$\vec{B} = 4\frac{\mu_0 I}{2\sqrt{2}\pi R}\hat{z} = \frac{\sqrt{2}\mu_0 I}{\pi R}\hat{z}. \tag{19}$$

(b) For an n-sided polygon, we can use the same approach as in part (a) except the angles in equation 17 will go from $\theta_1 = \frac{-\pi}{n}$ to $\theta_2 = \frac{\pi}{n}$ with $s = R$. As such, each side contributes:

$$B = \frac{\mu_0 I}{4\pi R}\left(\sin\left(\frac{\pi}{n}\right) - \sin\left(-\frac{\pi}{n}\right)\right) = \frac{\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{n}\right). \tag{20}$$

Since there are n sides to the polygon, the total $\vec{B}$ at the center becomes:

$$\vec{B} = \frac{n\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{n}\right)\hat{z}. \tag{21}$$

(c) In the limit of a circular loop, $n \to \infty$ and $\sin\left(\frac{\pi}{n}\right) \to \frac{\pi}{n}$, therefore equation 21 goes to:

$$\vec{B} = \lim_{n\to\infty}\frac{n\mu_0 I}{2\pi R}\sin\left(\frac{\pi}{n}\right)\hat{z} = \frac{n\mu_0 I}{2\pi R}\frac{\pi}{n}\hat{z} = \frac{\mu_0 I}{2R}\hat{z}, \tag{22}$$

which is in fact the solution we found in equation (5.38) (if you set $z = 0$).

5. **Griffiths Problem 5.09:** Find the magnetic field $\vec{B}$ at point P for each of the steady current configurations shown in Fig. 5.23 (below).

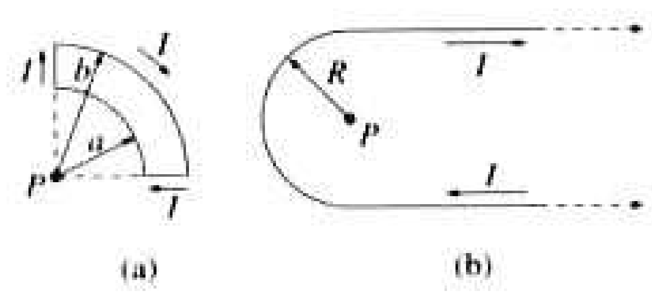


Figure 5.23

HINT: Since $\vec{B}$ is a vector field, direction must be stated in the correct answer. Also, for part (b), start by using the same general solution for the magnetic field due to a *segment* of straight uniform current in Example 5.5 you used to solve the Griffiths 5.08.

We are to find the magnetic field $\vec{B}$ due to two different current configurations. We will tackle both by using the Biot-Savart law the magnetic field due to a small segment of current I and length $d\vec{\ell}$ is

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{\ell} \times \hat{\mathfrak{r}}}{\mathfrak{r}^2}. \tag{23}$$

(a) For the curved segment of radius a all points on the wire are the same distance $\mathfrak{r}$ from P, so the field made by the segment is just

$$B_a = \frac{\mu_0 I}{4\pi} \frac{(\pi/2)a}{a^2} = \frac{\mu_0 I}{8a}. \tag{24}$$

This field points out of the page according to the right hand rule (**NOTE:** Recall $\vec{\mathfrak{r}}$ points from the current element to the location you are measuring the $\vec{B}$ field). The two straight segments produce zero field at the point P because $d\vec{\ell}$ and $\mathfrak{r}$ point in the same direction. The segment with radius b produces a field whose magnitude is $\mu_0 I/(8b)$ but is pointed into the page, so the total field is

$$B = \frac{\mu_0 I}{8}\left(\frac{1}{a} - \frac{1}{b}\right) \text{ (into the page);} \tag{25}$$

because $a < b$ the net field is positive, which is out of the page.

(b) The field due to a straight wire segments is, from Eq. (5.35) in the book,

$$B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1). \tag{26}$$

For the bottom wire, $\theta_1 = 0$ and $\theta_2 = \pi/2$ so the field is $B = \mu_0 I/(4\pi R)$; the right hand rule gives a direction into the page for the magnetic field. The field for the top straight wire is the same. The curved segment has field

$$B_{\text{curved}} = \frac{\mu_0 I}{4\pi}\frac{\pi R}{R^2}, \tag{27}$$

also into the page, so that the total field is

$$B = -\frac{\mu_0 I}{4R}\left(\frac{2}{\pi} + 1\right) \text{ (into the page)} \tag{28}$$

6. **Griffiths Problem 5.15:** Two long coaxial solenoids each carry current I, but in opposite directions, as shown in Fig. 5.42 (below). The inner solenoid (radius a) has n_1 turns per unit length, and the outer one (radius b) has n_2. Find $\vec{B}$ in each of the three regions: (i) inside the inner solenoid, (ii) between them, and (iii) outside both.

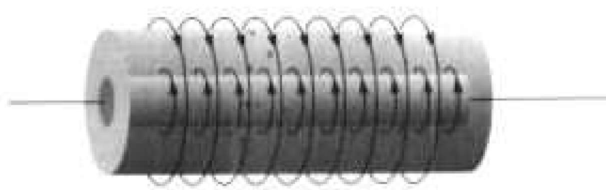
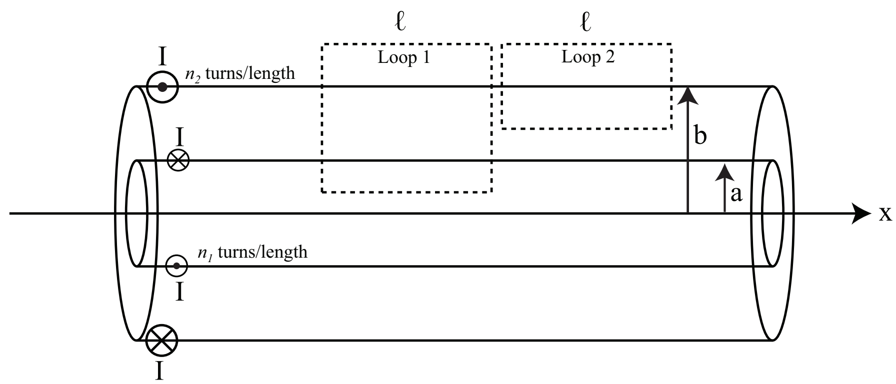


Figure 5.42

We start by noting there can be no radial component to $\vec{B}$, because, as argued in Example 5.9, if there were a radial component, reversing the direction of the current would switch this component. However, reversing the current is the same as rotating the solenoid 180 degrees, which should not alter a radial field. Therefore, any $\vec{B}$ field must be parallel to the axis of the solenoid. This helps us select Amperian loops which will be easiest to work with by only using segments parallel to or perpendicular to $\vec{B}$. The diagram below shows the solenoids and two Amperian loops we will use to find the field.



We will integrate counterclockwise around each loop.

(i) To determine the magnetic feld within the inner solenoid, I use "Loop 1". The current enclosed in Loop 1 is $I_{\text{enc}} = n_2\ell I - n_1\ell I = (n_2 - n_1)\ell I$. The integral of $\vec{B}$ around the loop is $\oint \vec{B}\cdot d\vec{\ell} = \ell B$, since the only segment of the wire on which the parallel component of the magnetic field is non-zero is the "bottom" segment in the solenoid (parallel to the $\hat{x}$. Putting these two pieces together gives $B = \mu_0(n_1 - n_2)I$ for the region $s < a$. With the current in the outer solenoid coming out of the page, its contribution to the magnetic field will be in the $+\hat{x}$ whereas the inner solenoid will contribute a component in the $-\hat{x}$ direction, therefore:

$$\vec{B} = \mu_0(n_1 - n_2)I\hat{x}. \tag{29}$$

(ii) To determine the magnetic field between the inner and outer solenoid, I use the amperian loop labelled "Loop 2." For Loop 2 the current enclosed is $I_{\text{enc}} = n_2 I$ and the integral of B is that same as before, so $B = \mu_0 n_2 I$, in the $-\hat{z}$ direction.
By the way, you can do this whole problem without Ampere's law if you use the fact that the solenoid has magnetic field $\mu_0 nI$ and just add the magnetic fields of the solenoids like vectors.

(iii) I didn't sketch the loop used, but since the magnetic field outside of a solenoid is zero, this is also the case here outside both solenoids.
(Thanks to Dr. Craig for providing the initial draft of this solution in LaTeX form.)