E&M Problem Set 7

Due Friday, Leap Day (February 29) at 4pm

SPECIAL NOTE: I could not find a symbol that exactly matches the book's script r for the separation vector. Instead I am using the following notation: $\vec{\mathfrak{r}} = \vec{r} - \vec{r}'$, $\mathfrak{r} = |\vec{\mathfrak{r}}|$ and $\hat{\mathfrak{r}} = \vec{\mathfrak{r}}/\mathfrak{r}$.

1. **Griffiths Problem 3.28:** In Example 3.9 we derived the exact potential for a spherical shell of radius R, which carries a surface charge $\sigma = k\cos\theta$.
   (a) Calculate the dipole moment of this charge distribution.
   (b) Find the approximate potential, at points far from the sphere, and compare the exact answer [$V(r,\theta) = \frac{kR^2}{3\epsilon_0}\frac{1}{r^2}\cos\theta$ $(r \geq R)$, Eq. 3.87]. What can you conclude about the higher multipoles?

2. **Griffiths Problem 3.33:** Show that the electric field of a ("pure") dipole [$\vec{E}_{dip}(r,\theta) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$, Eq. 3.103] can be written as
   $$\vec{E}_{dip}(r,\theta) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}(3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p})$$
   Also clarify why this expression is referred to as the coordinate-free form of the dipole electric field. **HINT:** It might be useful to work backward from the expression above back to equation 3.103. It is also helpful to note that $\vec{p} = (\vec{p}\cdot\hat{r})\hat{r} + (\vec{p}\cdot\hat{\theta})\hat{\theta}$.

3. **Griffiths Problem 3.40:** A thin insulating rod, running from $z = -a$ to $z = +a$, carries the indicates line charges. In each case, find the leading term in the multipole expansion of the potential
   (a) $\lambda = k\cos\left(\frac{\pi z}{2a}\right)$, where k is a constant.
   (b) $\lambda = k\sin\left(\frac{\pi z}{a}\right)$, where k is a constant.
   (c) $\lambda = k\cos\left(\frac{\pi z}{a}\right)$, where k is a constant.

   HINT: To know what is the leading term in the multipole expansion, it will be helpful to compute the total charge Q and if that is zero, to compute the dipole moment $\vec{p}$ and if that is zero, compute the quadropole moment. Also, consider what these linear charge densities λ actually mean in terms of where the charge is.

4. **Griffiths Problem 4.03:** According to equation 4.1 ($\vec{p} = \alpha\vec{E}$), the induced dipole moment of an atom, $\vec{p}$, is proportional to the external field, $\vec{E}$. This is a "rule of thumb," not a fundamental law, and it is easy to concoct exceptions — in theory. Suppose, for example, the charge density of the electron cloud were proportional to the distance from the center, out to radius R (*e.g.*- $\rho(r) = Ar$ where A is a constant with the right units). To what power of E would p be proportional to in that case? Find the condition on $\rho(r)$ such that Equation 4.1 will hold in the weak-field limit. **HINT:** Assume the electron cloud is spherically symmetric and determine a function for the electric field given the charge distribution mentioned. This "internal" electric field has to balance the external one when the nucleus is off center by some amount d. Understand Example 4.1 for guidance.

5. **Griffiths Problem 4.10 tweaked:** A sphere of radius R carries a polarization $\vec{P}(\vec{r}) = k\vec{r}$, where k is a constant and $\vec{r}$ is the vector from the center.

   (a) Calculate the bound charges σ_b and ρ_b.

   (b) What is the meaning behind these "bound charges"? Are they actual charges and if not, why do we bother to compute them? In other words, of what use are they?

   (c) Find the electric field inside and outside the sphere. **HINT:** Answering part (b) first might help you with your approach this problem. Also, the solution to Griffiths 2.12 which was on Problem Set 3, might help.

6. **Griffiths Problem 4.14 tweaked:** When you polarize a neutral dielectric, charge moves a bit, but the *total* remains zero. This fact should be reflected in the bound charges σ_b and ρ_b. Given the expressions for the bound charges from Equations 4.11 ($\sigma_b \equiv \vec{P} \cdot \hat{n}$) and 4.12 ($\rho_b \equiv -\vec{\nabla} \cdot \vec{P}$), develop an expression for the total charge and show that the total charge vanishes. What does this mean? **HINT:** If you want to do this painlessly, you'll want to exploit the divergence theorem.