E&M Problem Set 6

Due Friday, February 22 at 4pm

SPECIAL NOTE: I could not find a symbol that exactly matches the book's script r for the separation vector. Instead I am using the following notation: $\vec{\mathfrak{r}} = \vec{r} - \vec{r}'$, $\mathfrak{r} = |\vec{\mathfrak{r}}|$ and $\hat{\mathfrak{r}} = \vec{\mathfrak{r}}/\mathfrak{r}$.

Hyperbolic Functions: Since it will likely come in useful in cartesian boundary value problems, you should note that the hyperbolic functions are defined as

$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\text{and } \sinh x = \frac{e^x - e^{-x}}{2}.$$

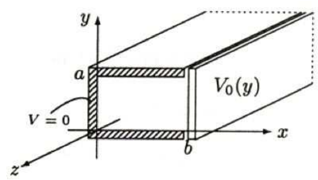
1. **Griffiths Problem 3.09 (variant):** A uniform line charge λ is places on an infinite straight wire, a distance d above a grounded conducting plane. (Lets say the wire runs parallel to the x-axis and directly above it, and the conducting plane is the xy plane.)

   (a) Examine Problem 2.47 in the textbook, which has solution $V(x, y, z) = \frac{\lambda}{4\pi\epsilon_0} \ln\left\{\frac{y^2+(z+d)^2}{y^2+(z-d)^2}\right\}$. Explain clearly why solving problem 2.47 is equivalent to solving the problem for the potential in the region above the plane. In the process, also explain clearly why it is *NOT* equivalent to solving for the potential below the plane!

   (b) Actually find the potential in the region above the plane! **HINT:** You did solve for the electric potential of a single infinite wire in Problem 2.22 which was on Problem Set 4. Use this information to quickly solve this problem.

   (c) Find the charge density σ induced on the conducting plane.

2. **Griffiths Problem 3.14:** A rectangular pipe, running parallel to the z-axis (from $-\infty$ to $+\infty$), has three grounded metal sides, at $y = 0$, $y = a$, and $x = 0$. The fourth side, at $x = b$, is maintained at a specified potential $V_0(y)$. See the Figure below.



NOTE: Figure is from Griffth's Instructor's Solution Manual, so it is Copright 1999 Prentice Hall.

   (a) Develop a general formula for the potential within the pipe. **Hint:** Looking at Example 3.4 in the textbook may give you some of the proper inspiration, but remember the boundary conditions on this problem are different!

   (b) Find the potential explicitly for the case $V_0(y) = V_0$ (a constant).

3. **Griffiths Problem 3.20:** Find the potential outside a *charged* metal sphere (charge Q, radius R) placed in an otherwise uniform electric field $\vec{E} = E_0\hat{z}$. Explain clearly where you are setting the zero of potential. **BIG HINT:** This is essentially a variant of Example 3.8. Consider that potential and the fact that potentials add like scalars to finish this problem with almost no calculations!

4. **Griffiths Problem 3.34 (tweaked):** A point charge q of mass m is released from rest at a distance a from an infinite grounded conducting plane.

   (a) Show that you can use conservation of energy to write an expression for the velocity v of the charge q at a height z above the infinite grounded conducting plane as:

   $$v = -\sqrt{\frac{q^2}{2m}\frac{1}{4\pi\epsilon_0}\left[\frac{1}{z} - \frac{1}{a}\right]} \tag{1}$$

   HINT: See section 3.2.3 of the book for an expression of the energy of a point charge above an infinite grounded conducting plane. Also note that you can safely ignore gravitational potential energy here since electric forces are much, much more powerful than gravitational forces here.

   (b) Using this expression for the total energy of the charge and the fact $v = \frac{dz}{dt}$, find an expression relating the time differential dt to the positional differential dz. Integrate both sides of this relationship to solve for the time the charge impacts on the conducting plane, t. [*Answer:* $\frac{\pi a}{q}\sqrt{2\pi\epsilon_0 ma}$]