E&M Problem Set 5

Due Wednesday, February 13 in lecture

Midterm #1: Our first Midterm Exam is scheduled for Friday, February 15. I am asking for this assignment early in order to allow you time to study.

SPECIAL NOTE: I could not find a symbol that exactly matches the book's script r for the separation vector. Instead I am using the following notation: $\vec{\mathfrak{r}} = \vec{r} - \vec{r}'$, $\mathfrak{r} = |\vec{\mathfrak{r}}|$ and $\hat{\mathfrak{r}} = \vec{\mathfrak{r}}/\mathfrak{r}$.

1. **Griffiths Problem 3.2:** In one sentence, justify **Earnshaw's Theorem:** *A charged particle cannot be held in a stable equilibrium by electrostatic forces alone.* As an example, consider the cubical arrangement of fixed charges in Figure 3.4 (below). It *looks*, off hand, as though a positive charge at the center would be suspended in midair, since it is repelled away from each corner. Where is the leak in this "electrostatic bottle"? [To harness nuclear fusion as a practical energy source it is necessary to heat a plasma (soup of charged particles) to fantastic temperatures −− so hot that contact would vaporize any ordinary pot. Earnshaw's Theorem says that electrostatic containment is also out of the question. Fortunately it *is* possible to confine a hot plasma *magnetically.*]

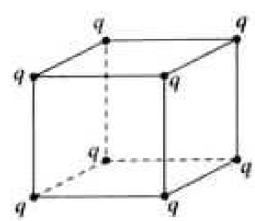
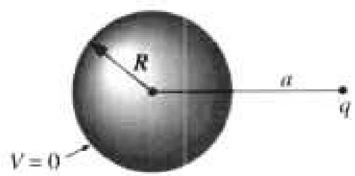


Figure 3.4

2. **Griffiths Problem 3.3:** Find the general solution ot Laplace's equation in spherical coordinates, for the case where V depends only on r. Do the same for cylindrical coordiantes, assuming V depends only on s.

3. **Griffiths Problem 3.7:** This problem asks you to essentially finish image problem laid out in Example 3.2. That example asked you to determine the potential outside a grounded conducting sphere with radius R with a point charge q located distance $a > R$ from the center of the sphere as shown in Figure 3.12 (reproduced below):



It argued that the corresponding *method of images* solution has a potential given by equation 3.17 (in part [a] below), but left it for this problem for you to verify that. Read that example carefully before starting this problem.

(a) Using the law of cosines, show that

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{\mathfrak{r}} + \frac{q'}{\mathfrak{r}'}\right)$$

(equation 3.17 from the textbook, describing the potential outside the sphere) can be written as follows:

$$V(r,\theta) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{r^2 + a^2 - 2ra\cos\theta}} + \frac{q}{\sqrt{R^2 + \left(\frac{ra}{R}\right)^2 - 2ra\cos\theta}}\right]$$

(b) Find the induced surface charge on the sphere, as a function of θ. Integrate this to get the total induced charge. (What *should* it be?)

(c) Calculate the energy of this configuration. **HINT:** Use the force given by equation 3.18 and the definition of work.

4. **Griffiths Problem 3.10 (tweaked):** Two semi-infinite grounded conducting planes meet at right angles. In the region betwen them, there is a point charge q, situated as shown in Figure 3.15 (below).

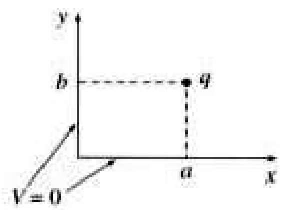


Figure 3.15

   (a) Set up the image configuration, and calculate the potential in this region. What charges do you need, and where should they be located?

   (b) What is the force on q?

   (c) How much work did it take to bring q in from infinity?

   (d) **(Extra Credit)** Suppose the planes met at some angle other than 90^o; would you still be able to solve the problem by the method of images? If not, for what particular angles *does* the method work?