**E&M Problem Set 4**

Due Friday, February 8 at 4PM

**SPECIAL NOTE:** I could not find a symbol that exactly matches the book's script r for the separation vector. Instead I am using the following notation: $\vec{\mathfrak{r}} = \vec{r} - \vec{r}'$, $\mathfrak{r} = |\vec{\mathfrak{r}}|$ and $\hat{\mathfrak{r}} = \vec{\mathfrak{r}}/\mathfrak{r}$.

1. **Griffiths Problem 2.22:** Find the potential a distance $s$ from an infinitely long straight wire that carries a uniform line charge density $\lambda$. Compute the gradient of your potential, and check that it yields the correct $\vec{E}$ field. **HINT:** You will need to choose a reference point a fixed distance $s_0$ from the wire for the potential. Be sure to explain why you need to do this. Also, for the sake of brevity, you may use the solution for the electric field of an infinitely long straight wire we worked out in lecture by citing it.

2. **Griffiths Problem 2.27:** Find the potential on the axis of a uniformly charged solid cylinder, a distance $z$ from the center. The length of the cylinder is $L$, its radius is $R$, and the charge density is $\rho$. Use your result to calculate the electric field at this point. (Assume that $z > \frac{L}{2}$, that is that the point you are measuring the potential at is outside the cylinder.) **SOME BIG HINTS:** It may be useful to use the potential above the axis of a flat disk with a constant surface charge density $\sigma$ that we derived in lecture. Also, we can relate surface and volume charge densities as $\sigma = \rho dz$ for a slab of thickness $dz$. If you use this, you better justify it. Finally, you will get a very ugly mess for the potential where at one point the relationship $\left(z + \frac{L}{2}\right)^2 - \left(z - \frac{L}{2}\right)^2 = 2zL$ might be helpful in simplifying things somewhat. If everything works, the final electric field on the $z$-axis works out to $E_z = \frac{\rho}{2\epsilon_0}\left[\sqrt{R^2 + (z - L/2)^2} - \sqrt{R^2 + (z + L/2)^2} + L\right]\hat{z}$.

3. **Griffiths Problem 2.32 (tweaked):** Find the energy stored in a uniformly charge solid sphere of radius $R$ and charge $q$. Do it two different ways:

   (a) Use equation 2.43 from the textbook
   $$W = \frac{1}{2}\int \rho V \, d\tau.$$
   We found the potential $V$ that is appropriate here when we did problem 2.21 in lecture.

   (b) Use equation 2.45 from the textbook
   $$W = \frac{\epsilon_0}{2}\int_{allspace} E^2 d\tau.$$
   Don't forget to integrate over *all space.*

4. **Griffiths Problem 2.35:** A metal sphere of radius $R$, carrying charge $q$, is surrounded by a thick concentric metel shell (inner radius $a$, outer radius $b$, as in Figure 2.48). The shell carries no net charge.

   (a) Find the surface charge $\sigma$ at $R$, at $a$, and at $b$.

   (b) Find the potential at the center, using infinity as the reference point.

   (c) Now the outer surface is touched to a grounding wire, which lowers its potential to zero (same as infinity). How do your answers to (a) and (b) change?

5. **Griffiths Problem 2.37:** Two large metal plates (each of area $A$) are held a distance $d$ apart. Suppose we put a charge $Q$ on each plate; what is the electrostatic pressure on the plates? **HINT:** Read Section 2.5.3.

6. **Griffiths Problem 2.43:** Find the net force that the southern hemisphere of a uniformly charged sphere exerts on the northern hemisphere. Express your answer in terms of the radius $R$ and the total charge $Q$. [*Answer*: $\frac{1}{4\pi\epsilon_0}\frac{3Q^2}{16R^2}$ ] **HINT:** You will need the electric field inside a uniformly charged sphere from Problem 2.12, which we discussed in lecture. Compute a force per unit volume on the northern hemisphere based on the that electric field, then work out the solution.