

### E&M Problem Set 1

Due Tuesday, January 22 at 4PM

**SPECIAL NOTE:** I could not find a symbol that exactly matches the book's script  $r$  for the separation vector. Instead I am using the following notation:  $\vec{\mathbf{r}} = \vec{r} - \vec{r}'$ ,  $\mathbf{r} = |\vec{\mathbf{r}}|$  and  $\hat{\mathbf{r}} = \vec{\mathbf{r}}/\mathbf{r}$ .

1. **Griffiths Problem 1.9:** (Chapter 1, Problem 9) Find the transformation matrix  $R$  that describes a rotation by  $120^\circ$  about an axis from the origin through the point  $(1,1,1)$ . The rotation is clockwise as you look down the axis toward the origin.
2. **Griffiths Problem 1.13:** Let  $\vec{\mathbf{r}}$  be the separation vector from a fixed point  $(x', y', z')$  to the point  $(x, y, z)$ , and let  $\mathbf{r}$  be its length. Show that
  - (a)  $\vec{\nabla}(\mathbf{r}^2) = 2\vec{\mathbf{r}}$
  - (b)  $\vec{\nabla}\left(\frac{1}{\mathbf{r}}\right) = -\frac{\hat{\mathbf{r}}}{\mathbf{r}^2}$
  - (c) What is the *general* formula for  $\vec{\nabla}(\mathbf{r}^n)$ ?
3. **Griffiths Problem 1.16:** Sketch the vector function  $\vec{v} = \frac{\hat{r}}{r^2}$  and compute its divergence. The answer may surprise you...can you explain it?
4. **Griffiths Problem 1.26:** Prove the divergence of a curl is always zero. *Check* it for function  $\vec{v}_a = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$  (from Problem 1.15).
5. **Griffiths Problem 1.37:** Express the unit vectors  $\hat{r}, \hat{\theta}, \hat{\phi}$  in terms of  $\hat{x}, \hat{y}, \hat{z}$  (that is, derive Eqn. 1.64). Check your answers several ways (does  $\hat{r} \cdot \hat{r} = 1$ ,  $\hat{\theta} \cdot \hat{\phi} = 0$ ,  $\hat{r} \times \hat{\theta} = \hat{\phi}$  ...). Also work out the inverse formulas, giving  $\hat{x}, \hat{y}, \hat{z}$  in terms of  $\hat{r}, \hat{\theta}, \hat{\phi}$ .