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Physics 370 Mid-Term #2 Study Guide
Date of Exam: Friday, March 28

Rules:
1. Closed Book and Closed Note. A formula sheet will be provided.
2. You will be allowed the entire 70 minutes for the exam, although I will be aiming for it to be 60 minutes in length.
3. You will be allowed a calculator (although I will avoid problems where it will be required).
4. If you are stuck because you can't remember a formula or an indefinite integral, I assure you that you can ask me… I am not big on formula memorization.

Topics on this Exam (with some notes):

You can expect the problems on this mid-term exam to focus on the concepts on problem sets 5 through 9. Of course, since like almost any academic field, basic principles build into advanced concepts. I do expect knowledge of the material covered on MidTerm #1.

The problems will be on the order of difficulty of the problem set problems, although I will avoid any integrands that are not terribly obvious to resolve. **There will be some conceptual problems, so be sure you have reviewed concepts as well as mathematical techniques.**

1. Special Techniques In Electrostatics (Chapter 3)

A. **Solutions to Laplace's Equation**
   1. Know the properties of a solution for the electric potential in a region with no electric charges (*i.e.* – the properties of solutions to Laplace's equation).
   2. Know the meaning behind the first and second uniqueness theorems.

B. **Method of Images**
   1. Know why the method of images works and what are the restrictions on the locations of the "image" charges as well as the rules for where you place the image charges in order to solve a given problem.
   2. Why is it that solving a completely different problem using "image" charges can solve an electrostatics problem correctly?

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C. **Separation of Variables**

1. Know in what situations it is possible to use the technique of separation of variables.
2. You should know the general form of the solutions of problems in the following coordinate systems when separation of variables is possible.
    a. Cartesian coordinates
    b. Spherical coordinates (with no azimuthal dependence). **NOTE:** Should you need Legendre Polynomials, a table will be provided.

D. **Multipole Expansion**

1. Know what is meant by an electric dipole. Specifically, know what form of charge distribution leads to suppression of the monopole term and dominance of the dipole term in the multipole expansion of the electric potential.
2. Know what charge distributions lead to dominant electric monopole, dipole, and quadrupole terms.
3. In which situations is it appropriate to employ the technique of multipole expansion.

2. Electric Fields in Matter (Only Chapter 4.1 – 4.3, skip 4.1.3)

A. Polarization in Dielectrics (aka Insulators)

1. Know the definition of a dielectric (versus a conductor).
2. Know how dielectrics respond to the application of an external electric field and what is meant by the polarization of a dielectric.
3. Know what is meant by the atomic polarizability of a dielectric.

B. Field of a Polarized Object

1. Know what is meant by "bound charges" and how an electrically neutral dielectric can have "bound charges".
2. Know <u>why</u> the bound surface charge and bound volume charge can be expressed by

$$\sigma_b = \vec{P}\cdot\hat{n}$$
$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

That is, know the <u>meaning</u> behind the equations… what is physically going on in the dielectric?

C. Electric Displacement

1. Know what the form is of Gauss's Law within a dielectric.
2. Given a polarization of a material, be able to solve for the electric field **E** and the electric displacement **D.**

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3. Magnetostatics (Chapter 5)

A. The Lorentz Force Law

1. Know the Lorentz force law and the situations in which it applies. More importantly, know the situations in which it is helpful.
2. **Remember, if you write with your right hand to drop the pencil before applying the right-hand-rule for determining curl direction.**
3. Know what work can be done by magnetic forces…
4. Consider Problem 5.3 that I assigned in the homework. Why do we know magnetic forces applied to moving electric charges in the absence of an electric field always lead to "circular" motions.

B. Currents

1. Know the definitions of line current **I**, surface current **K**, and volume current **J**.
2. Know the various expressions for current (e.g. $\mathbf{I} = \lambda\, \mathbf{v}$, $d\mathbf{I} = \lambda\, d\mathbf{l}'$, $d\mathbf{J} = \rho\, d\tau'$. Basically equations 5.14 through 5.27).
3. Know <u>why</u> a surface (volume) current can be viewed as a current per unit length (area) perpendicular to the flow of the current.
4. Know what is meant by the continuity equation (5.29) and more importantly, its meaning.
5. Know what is meant by a "steady current" and why in ***magnetostatics***, we know $\nabla \bullet \mathbf{J} = 0$.

C. The Biot-Savart Law

1. Know what the Biot-Savart law is and situations in which it can be applied.
2. Know how to deal with linear currents and (semi-) circular current loops in terms of the Biot-Savart law.

D. Divergence and Curl of B

1. Know what the divergence and curl of a magnetostatic field are.
2. Know <u>what</u> it physics means for the magnetic field to have a divergence of zero.
3. Know <u>what</u> it physically means for the magnetic field to have a non-zero curl.
4. **Ampère's Law and its applications**
   - **a.** Know the general technique for applying Ampère's law to a given situation as outlined in class.
   - **b.** Know what symmetries allow easy application of Ampère's law.

E. Magnetic Vector Potential

1. Why is it possible to wrote a magnetic field **B** in terms of the curl of a magnetic vector potential **A**?
2. Know how to compute the magnetic vector potential **A** for a given distribution of currents.
3. Know how to compute the magnetic field **B** corresponding to a given magnetic vector potential **A**.

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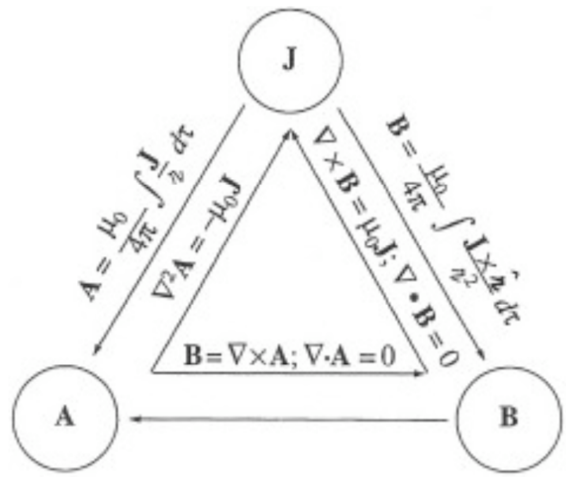
F. Magnetostatic Boundary Conditions

1. Know what is meant by magnetostatic boundary conditions. Specifically how the magnetic field changes when crossing a surface current and how that can be related to the magnetic vector potential in that region.
2. The following diagram is worth committing to memory, at least conceptually. It is an illustration showing how the volume current density **J**, magnetic field **B**, and magnetic vector potential **A** are all related and how to go from one to another.



This is **Figure 5.48** from *Introduction to Electrodynamics,* by D.J. Griffiths (Prentice-Hall) © 1999 Prentice-Hall Inc.

G. Multipole Expansion of the Magnetic Vector Potential

1. Know what is meant by an magnetic dipole and how it is similar to and different from an electric dipole.
2. What happens to the monopole term in the magnetic vector potential?
3. In which situations is it appropriate to employ the technique of multipole expansion of the magnetic vector potential.

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Potentially Useful Relationships (Not a complete list)

Del (cartesian): $\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

Gradient (cartesian): $\vec{\nabla}V = \frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}$

(spherical): $\vec{\nabla}V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$

Divergence (cartesian): $\vec{\nabla}\cdot\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

(spherical): $\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\, v_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$

Curl (cartesian):

$$\vec{\nabla}\times\vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

$$= \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{z}$$

(spherical):

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}\left(\sin\theta\, v_\phi\right) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{r}$$

$$+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}\left(r\, v_\phi\right)\right]\hat{\theta}$$

$$+\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial \theta}\right]\hat{\phi}$$

Laplacian (cartesian): $\vec{\nabla}^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Potentially Useful Constants or units

permittivity of free space: $\epsilon_0 = 8.85 \times 10^{-12} C^2/N \cdot m^2$

permeability of free space: $\mu_0 = 4\pi \times 10^{-7} N/A^2$

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Potentially Useful Relationships (Continued)

Separation Vector:	$\vec{\mathfrak{r}}$	$= \lvert \vec{r} - \vec{r}' \rvert$
Electric Field:	$\vec{E}(\vec{r})$	$= \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\rho(\vec{r}')}{\mathfrak{r}^2} \hat{\mathfrak{r}} d\tau'$
Gauss' Law:	$\oint_S \vec{E} \cdot d\vec{a}$	$= \frac{Q_{enc}}{\epsilon_0}$
	$\vec{\nabla} \cdot \vec{E}$	$= \frac{\rho}{\epsilon_0}$
Electric Potential:	$V(\vec{r})$	$\equiv -\frac{1}{4\pi\epsilon_0} \int_{O}^{\vec{r}} \vec{E} \cdot d\vec{l}$
Field vs. Potential:	$\vec{E}$	$= -\vec{\nabla} V$
Poisson's Equation:	$\vec{\nabla}^2 V$	$= -\frac{\rho}{\epsilon_0}$
Laplace's Equation:	$\vec{\nabla}^2 V$	$= 0$
Multipole Expansion of V	$V(r,\theta)$	$= \sum_{\ell=0}^{\infty} \left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell-1}} \right) P_n(\cos\theta') \rho d\tau'$
Electric Dipole:	$\vec{p}$	$= \int \vec{r}' \rho d\tau'$
Dipole Electric Potential Term:	V_{dip}	$= \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0 r^2}$
Dipole Magnetic Field:	$\vec{E}_{dip}(\vec{r})$	$= \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$
Bound Charges:	σ_b	$= \vec{R} \cdot \hat{n}$
	ρ_b	$= -\vec{\nabla} \cdot \vec{P}$
Gauss's Law for Dielectrics:	$\vec{\nabla} \cdot \vec{D}$	$= \rho_f$
	$\oint \vec{D} \cdot d\vec{a}$	$= Q_{f_{enc}}$
Electric Displacement:	$\vec{D}$	$= \epsilon_0 \vec{E} + \vec{P}$
Lortenz's Law:	$\vec{F}_{mag}$	$= Q(\vec{v} \times \vec{B})$
	$\vec{F}_{mag}$	$= I(d\vec{\ell} \times \vec{B})$
Continuity Equation:	$\vec{\nabla} \cdot \vec{J}$	$= -\frac{\partial \rho}{\partial t}$
Biot-Savart Law:	$\vec{B}(\vec{r})$	$= \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \hat{\mathfrak{r}}}{\mathfrak{r}^2} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\ell} \times \hat{\mathfrak{r}}}{\mathfrak{r}^2}$
Ampère's Law:	$\vec{\nabla} \times \vec{B}$	$= \mu_0 \vec{J}$
	$\oint \vec{B} \cdot d\vec{\ell}$	$= \mu_0 I_{enc}$
Magnetic Vector Potential $\vec{A}$:	$\vec{B}$	$= \vec{\nabla} \times \vec{A}$
	$\vec{A}(\vec{r})$	$= \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{\mathfrak{r}} d\tau'$
Magnetic Dipole moment:	$\vec{m}$	$\equiv I \int d\vec{a} = I\vec{a}$
Magnetic Dipole Vector Potential:	$\vec{A}_{dip}(\vec{r})$	$= \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$
Dipole Magnetic Field:	$\vec{B}_{dip}(\vec{r})$	$= \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})$