Physics 350 Lab 13: Convolutions

Objective: In this lab you will use convolution to manipulate functions and image, and practice the image manipulation skills you learned last week.

Background: We have two ways of thinking of the convolution of a function $f(x)$ and another function $k(x)$ often called the kernel. You can accomplish this convolution in IDL be creating arrays (1-D or 2-D) for f and k and then calling the `convol` command as follows (here I am assigning the convolution to a variable called h)

```
h=convol(f,k,/EDGE_WRAP,/NORM)
```

You can look up what the options do in the help for IDL.

One way to think about the convolution $f * k$ is that each point in the function f is replaced by a weighted average of nearby point. For example, consider the"boxcar" function like the one in this week's homework, homework,

$$f(x) = \begin{cases} 1, & 0 < x < 2, \\ 0, & \text{otherwise}; \end{cases} \tag{1}$$

you can plot it by doing this:

```
N=20L   ; number of points in the x direction
xMax = 10.
x = xMax*findgen(N)/(N-1)  ; generates N points from 0 to xMax
f=boxcar_lab(x,Max=2)   ; make sure you have download the lab
                        ; supplement with this file!
plot,x,f, PSYM=1, YRANGE=[0,2]
```

Now suppose we convolve this with a kernel $g(x)$ that is

```
g = [1,0,1]
```

The effect of this will be explained on the board at the beginning of lab.

Procedure:

1. **1D Convolution in real space:**
   (a) Find a kernel g with three elements that takes the derivative of a function. **Hint:** The kernel will be something like `g=[ ?, 0, ?]` You figure out what would go in each of the missing spots; you won't put the same number into each spot, and remember a derivative is a difference.
   (b) Test your derivative kernel by convolving it with a boxcar, a triangle and a sine-like function (functions are all in the lab supplement)
   (c) Find a kernel that takes the second derivative. Hint: It will be something like `g=[?, 2, ?]` and in this case the missing spots are filled with the same number.
   (d) Test your second derivative kernel by convolving it with a boxcar, a triangle, and a sine-like function.
   (e) **WHAT TO TURN IN FOR THIS PART:** IDL code that does the convolution, plots the convoluion of each of the three functions with each of your kernels (so 6 plot commands total), and an a short explanation of how you know your kernel is right that refers to the plots. YOU SHOULD NOT TURN IN THE PLOTS, JUST THE CODE!

2. **2D Convolution: Smoothing images in frequency space:** One way to smooth an image is to imagine replacing the value of the image at each pixel with the average of several of its neighbors. As an integral you would write this averaging as

$$I_{smooth}(x, y) = \int_{all\ points\ in\ image} I(x', y') f_R(x - x', y - x') dx' dy', \tag{2}$$

   where f_R is called a filter, with radius R, given by

$$f_R(x, y) = \begin{cases} 1, & \sqrt{x^2 + y^2} < R \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

   Download `Lab13Supplement.zip`, which contains the image you need for today, `capitol_small.png` and the function `round_filt.pro`
   (a) The function `round_filt` makes a filter, in real space, for an image. Execute this IDL command to make the filter:

```
radius=10.0
filt = round_filt(radius)
```

then display `filt` using `tvscl`. Repeat this for a few different values of radius until you understand what the filter looks like. You should see a round quarter circle in each of the corners of the image. Why does it look that way even though we centered the filter on the origin?

(b) Try using this filter directly in position space by multiplying the image `capitol_small.png` with `filt` and displaying the result.

(c) Now we will try convolving this filter with the image using the Fourier transform rather than using the convolution in Eq. (2) directly.

i. Calculate the FFT of `filt`, and display the absolute value of the transform using `tvscl`

ii. Transform the image, multiply the transform of the image by the transform of the filter.

iii. Use the inverse transform to obtain the filtered image, display it and describe how it has changed.

(d) The edges of the smoothed image should look somewhat odd. Why?