

Physics 350 Lab 7: Rotational Transforms

Objective: The idea of a rotational transform embodied in a 3x3 matrix was introduced earlier this week in lecture. Today you will build a matrix for performing a rotational transform around an axis in a somewhat arbitrarily chosen direction.

Lab Requirements:

The supplemental *Maple* worksheet for this lab (`lab07supplement.mw`) can be found online on the class website in the Electronic Handouts area. It demonstrates the syntax for doing matrix algebra on the computer so that you can perform eigenvalue problems.

Procedure: First, convince yourself that two successive rotations about the z -axis through angles ϕ_1 and ϕ_2 is equivalent to a single matrix operation (rotation) through angle $(\phi_1 + \phi_2)$. You'll need to recall Euler's formula, which can be used to derive the useful expressions

$$\begin{aligned}\sin(\phi_1 + \phi_2) &= \sin \phi_1 \cos \phi_2 + \cos \phi_1 \sin \phi_2 \\ \cos(\phi_1 + \phi_2) &= \cos \phi_1 \cos \phi_2 - \sin \phi_1 \sin \phi_2.\end{aligned}$$

This idea of successive transformations carries over to multiple operations that involve several matrices multiplied together **in the appropriate order**. You'll use this in what follows.

1. Identify (and call it R) the 3x3 matrix for a rotation through angle ϕ about the positive z axis. Such that for example a rotation of $\phi=90^\circ$ would bring a vector that is initially pointed along the x axis to point along the y axis.
2. Identify (and enter into a Maple worksheet) the 3x3 matrix (call it M) for the transformation which brings the point $(1,1,1)$ to the point $(0,0,\sqrt{3})$ via a counterclockwise rotation about the line from the point $(1,-1,0)$ to the origin (looking toward the origin). The rotation is through angle α , where $\cos(\alpha) = 1/\sqrt{3}$. [**Big Hint:** Think of this as two successive rotations, one rotating point $(1,1,1)$ onto the yz plane by rotating about the z axis, then another rotation to rotate the point onto the z axis.]
 - a. In addition to determining what M is, have Maple compute M^{-1} .
3. Find the matrix (call it S) that rotates counterclockwise by an angle ϕ about the line segment from the point $(1,1,1)$ to the origin (again, looking toward the origin). *Hint:* You should show that the rotation you desire is the same as $M^{-1}RM$. Once you have this matrix as a function of ϕ , establish the following.
 - a. Choose $\phi = 120^\circ$ and then operate separately on the unit vectors $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$. Establish where each of them goes. Do your results make sense? This represents a consistency check! Can you see why?
 - b. For $\phi = 60^\circ$, where does the point $(1,2,4)$ go?
 - c. For $\phi = 90^\circ$, where does the point $(1,1,0)$ go?