Physics 350 Lab 5: Fourier Series with Maple

Objective: The first objective is to write a somewhat more complicated group of functions than we have so far in *Maple* using the `proc` (stands for "procedure") command. The second is to explore the properties of the Fourier sine and cosine series. In particular you should be able to decide from the symmetry of a function about the center of the interval you are examining whether it can be represented by a sine series only, a cosine series only, or whether a full series must be used.

Recall in class we discussed the Fourier series as an approach to solving damped driven harmonic oscillators. The assumption is that any function that is periodic over interval 0 to T can be written:

$$f(t) = \sum_{n=0}^{\infty} A_n \cos\left(\frac{2\pi n}{T} t\right) + B_n \sin\left(\frac{2\pi n}{T} t\right)$$

where the coefficients for the Fourier cosine series can be written:

$$A_n = \frac{\int_0^T f(t)\cos\left(\frac{2\pi n}{T} t\right) dt}{\int_0^T \cos\left(\frac{2\pi n}{T} t\right)\cos\left(\frac{2\pi n}{T} t\right) dt} = \begin{Bmatrix} \frac{1}{T} & \text{for } n = 0 \\ \frac{2}{T} & \text{for } n > 0 \end{Bmatrix} \int_0^T f(t)\cos\left(\frac{2\pi n}{T} t\right) dt$$

and the coefficients for the Fourier sine series can be written:

$$B_n = \frac{\int_0^T f(t)\sin\left(\frac{2\pi n}{T} t\right) dt}{\int_0^T \sin\left(\frac{2\pi n}{T} t\right)\sin\left(\frac{2\pi n}{T} t\right) dt} = \frac{2}{T}\int_0^T f(t)\sin\left(\frac{2\pi n}{T} t\right) dt$$

Obviously these integrals can be quite tedious to resolve by hand for different n, so it is easier for us to let *Maple* handle the tedium instead.

Lab Requirements:
The supplemental *Maple* worksheet for this lab (`lab05supplement.mw`) can be found online on the class website in the Electronic Handouts area. You should download this worksheet first thing upon arriving in lab and review it in preparation for the lab.

Procedure: When you are finished with this exercise, either turn in a printout of the worksheet **with your name and lab number up top** or email that worksheet to your instructor. **Save often, as *Maple* is not always a stable environment.**

Prepare a *Maple* worksheet to carry out the following exercises:

1. Since you will need to fit Fourier cosine series later in this lab, it would be a good idea to start a new maple worksheet and copy the functions `c`, `A`, `cosineFS` and `cosineFP` from the lab supplement.

2. A Fourier sine series for a function *f(x)* is defined to be

$$f(x) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{2\pi n}{L} x\right)$$

where B_n is computed as shown above.

   a. Write functions in *Maple* to implement the Fourier sine series expansion of a function. In other words, write functions `s`, `B`, `sineFS` and `sineFP` in *Maple* similar to the functions `c`, `A`, `cosineFS` and `cosineFP` in the lab supplement.

   b. Find the Fourier cosine series and the Fourier sine series expansion for the function $f=x-L/2$. As in the supplement you may set $L=1$. For the sine series make an animated graph like those in the supplement and describe in words how the "goodness" of the sine series approximation changes as you change n.

3. The full Fourier series for a function is the sum of its cosine series and its sine series. For some functions the cosine series is zero and the sine series is not, for others the sine series is zero and the cosine series is not, and for other functions both the sine and cosine series are non-zero. In this problem you explore what properties of a periodic function determine whether it can be represented by a sine series, a cosine series, or requires a full Fourier series.

   a. For each of the functions below
      i. Plot the function for x=0 to L
      ii. Determine whether only the sine series is nonzero, only the cosine series is nonzero or both are nonzero.

      The functions are:
      - $x(L-x)$
      - $(x - L/2)^3$
      - $|x - L/2|$
      - $x = -1$ for $x<L/2$, $x=1$ for $x>L/2$

   b. Describe what features of a function seem to be related to which of the series is nonzero.