Physics 350 Lab 4: Differential Equations with Maple

Objective: This lab introduces you to the ordinary differential equation solving capabilities of Maple. You will also use the Maple command `animate` to explore the features of solutions to differential equations.

Lab Requirements:
The supplemental *Maple* worksheet for this lab (`lab04supplement.mw`) can be found online on the class website in the Electronic Handouts area. You should download this worksheet first thing upon arriving in lab and review it in preparation for the lab.

Procedure: When you are finished with this exercise, either turn in a printout of the worksheet **with your name and lab number up top** or email that worksheet to your instructor. **Save often, as *Maple* is not always a stable environment.**

Prepare a *Maple* worksheet to carry out the following exercises:

1. The equation of motion for a damped simple harmonic oscillator, written in dimensionless form, is
$$\frac{d^2x}{d\eta^2} + B\frac{dx}{d\eta} + 4\pi^2 x = 0$$
where $\eta = t/T$ is the time written as a fraction of a period, and *B* measures the size of the damping. Assume initial conditions *x(0)=1* and *v(0)=0*.
    a. Solve this differential equation in Maple using `dsolve`.
    b. Make an animated plot of the solutions in which the horizontal axis goes from 0 to five periods, and in which *B* changes from 0 to 20. Make your animation with 50 frames.
    c. As you change *B* you should see two very different behaviors for the solution. Describe each, and explain physically why each solution has the form it does. You might find it helpful to work out the solution to the differential equation above with pencil and paper to aid in you interpretation.

2. The equation of motion for a driven harmonic oscillator, written in the same dimensionless form as above, is
$$\frac{d^2x}{d\eta^2} + B\frac{dx}{d\eta} + 4\pi^2 x = \frac{4\pi^2 f}{\omega^2}\cos(\omega_d T\eta)$$
where ω_d is the driving frequency and *f* is the amplitude of the driving force. For this problem, assume *B*=1/20, and that $\frac{4\pi^2 f}{\omega^2} = 26.38$. Also assume initial conditions *x(0)=0* and *v(0)=0*.
    a. Solve this differential equation.
    b. Make an animated plot of the solutions, with $\cos(\omega_d T\eta)$ also on the same plot, in which the horizontal axis goes from 0 to 25 periods of the undamped oscillator, and in which $\omega_d T$ changes

from 0.2 to 10; the y-axis limits should be ±5. Make your animation with 50 frames, and make each plot with 100 points.

c. Describe how the amplitude of the response (in other words the solution to the differential equation) depends on the driving frequency.

d. Describe how the phase of the response (in other words the displacement of the peaks of the oscillation) depends on the driving frequency.

e. You should have noticed that there is a special frequency at which the solution changes fairly drastically (both in phase and in amplitude). How does that frequency seem to be related to the natural frequency of the oscillator?