

Physics 350 Lab 3: Spherical Coordinates

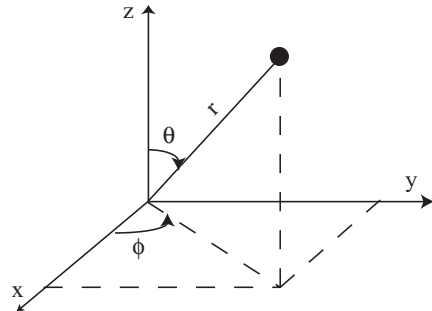
Objective:

Some physical problems are much more manageable if we exploit the symmetries in the problem rather than trying to do the problem in Cartesian xyz coordinates, it often makes more sense to use polar, cylindrical, or spherical coordinates. For example, the hydrogen nucleus, consisting of a single proton, is a spherically symmetric system. And since any interaction of a hydrogen nucleus with an electron (assuming the electron is not within the nucleus) will involve Coulomb's law, which depends only on distance between the electron and proton, so it makes no sense to solve the problem in Cartesian coordinates when the problem just screams out "I'm spherically symmetric."

In this exercise we will be focusing on doing some basic physics problems that have spherical symmetry and therefore lend themselves to being approached in spherical coordinates. Recall (from *Boas*) that the spherical coordinates are typically defined as:

$$\begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} & x &= r \sin \theta \cos \phi \\ \theta &= \cos^{-1} \frac{z}{r} & y &= r \sin \theta \sin \phi \\ \phi &= \tan^{-1} \frac{y}{x} & z &= r \cos \theta \end{aligned}$$

Maple uses a spherical coordinate system defined in this manner. Although as a warning, many texts will interchange the symbols for θ and ϕ (e.g. such that ϕ is the angle of the radial vector from the z axis).



Lab Requirements:

The supplemental *Maple* worksheet for this lab (`lab03supplement.mw`) can be found online on the class website in the Electronic Handouts area. You should download this worksheet first thing upon arriving in lab. It shows how to plot some three-dimensional functions in spherical coordinates. In particular, it uses the probability density¹ for a specific hydrogen atom orbital. For today's exercise, you will do similar calculations and prepare similar plots but for different orbitals.

¹ The probability density is the same as the modulus squared of the wave function.

Procedure:

When you are finished with this exercise, either turn in a printout of the worksheet **with your name and lab number up top** or email that worksheet to your instructor. **Save often, as Maple is not always a stable environment.**

Prepare a *Maple* worksheet to carry out the following exercises:

1. Let us begin by defining

$$P(r, \theta, \phi) = \frac{1}{39366\pi} r^4 e^{-2r/3} (3\cos^2 \theta - 1)^2$$

P is different from the function discussed on the supplement sheet as it describes a different hydrogen atom state.

- Convert this density P from spherical coordinates to Cartesian xyz coordinates, and make a contour plot showing a “slice” through this function in the xz plane. **Hint:** When entering the function, use the predefined constant `Pi` rather than `pi` (which is treated in *Maple* as a variable represented by the Greek letter π rather than the constant whose value is close to 3.14159...).
- Make two different three-dimensional plots of surfaces of constant probability for this function (using `implicitplot3d` as was done on the supplement sheet). Choose two different values for the contour level, and be sure that you plot a large enough volume to see the entire surface.
- Calculate the probability of finding the electron in a sphere of radius R , and plot the result as a function of R . Check and make sure that the limit as $R \rightarrow \infty$ makes sense. **Hint:** Again, When setting up limits, be sure to use the predefined constant `Pi` rather than `pi`.
- Calculate the probabilities that the electron would be found in the region $\theta < \pi/2$, the region $\theta < \pi/3$, and the region $\theta < \pi/4$. Use `evalf` to get the answers as floating point numbers.

2. Now consider a different hydrogen atom orbital, with probability density

$$P(r, \theta, \phi) = \frac{5}{2048\pi} \left(1 - \frac{r}{4} + \frac{r^2}{80}\right)^2 r^2 e^{-r/2} \sin^2 \theta.$$

Calculate the probability that the electron would be found in the regions (again, use `evalf` to get the answers as floating point numbers).

- $r < 1$,
- $1 < r < 2$,
- $r > 10$.
- Make some plots (both 3 and 2-D) of the probability versus radius to get a better feel for the actual shape of this probability density field.