Physics 350 Lab 2: Taylor's Series and Complex Arithmetic

Objective: This exercise is meant to give you some exposure to *Maple*'s ability to simplify the computation of Taylor's Series expansions of functions as well as allowing for the much quicker (although maybe not as educational) evaluation of arithmetic statements involving complex numbers.

Now, you have been doing plenty of complex arithmetic recently, so that aspect of this lab doesn't require much of an introduction. However, most of you have probably not seen Taylor's Series for a spell, so here's a quick review.

Taylor's series are a way to approximate any differentiable function as a infinite series. There is a quick review in Section 1.12 of the *Boas* text, but in essence, the Taylor's series approximation of any function of *x* about *x=a* can be written as:

$$f(x) = f(a) + (x-a)f'(x) + \frac{1}{2!}(x-a)^2 f''(x) + \frac{1}{3!}(x-a)^3 f'''(x) + \cdots$$

This approximation will be most accurate for values of *x* near *a* and generally degrades as you move to more removed values of *x*. Notice the Taylor's series is just a sequence of derivatives and factorials. Needless to say, it is not usually <u>hard</u> to write a Taylor's series expansion of a function… but it can be tedious, especially if you desire to have a lot of terms. This is where *Maple* can be useful, allowing the quick determination of Taylor's series expansion of any function.

Lab Requirements:
The supplemental *Maple* worksheet for this lab (`lab03supplement.mw`) can be found online on the class website in the Electronic Handouts area. You should download this worksheet first thing upon arriving in lab.

Procedure: After you review the *Maple* worksheet (`lab02supplement.mw`), create your own Maple worksheet that answers the following questions.

Part I) Taylor's Series in *Maple*

1. Calculate the Taylor series of the function e^{it} about $t = 0$, up to 15th order (Use `convert` to get rid of the $O(t^{15})$ term at the end.)
   a. Also calculate the Taylor series of *sin t* and *cos t* to the same order.
   b. Now verify Euler's formula by showing that the Taylor series for e^{it} is the same as the Taylor series for *cos t* + *i sin t*. (One good way to do this is to subtract the series from each other, have Maple simplify the result, and show that it is zero.)

2. Calculate the Taylor series for *cos x* about $x = 0$, going up to 10th order. Make a plot showing both *cos x* and this Taylor expansion over the range $x = 0$ to 5.

3. As you can see, the Taylor series for *cos x* agrees very well with the true function near $x = 0$, but the agreement gets worse as x gets larger. Have Maple find the point where the disagreement between *cos x* and its Taylor series is 0.1. (You can do this by using `solve`, but a faster way is to use `fsolve`. `solve` will try unsuccessfully to solve the equation algebraically. After Maple has realized this, it will find an approximate solution numerically. `fsolve` will skip the first step and just solve numerically right away.)

   This gives a rough idea of where the Taylor series starts to "go bad." Of course, if we'd calculated the Taylor series to higher order, the approximation wouldn't go bad until a larger value of x.

Part II) Complex Number Arithmetic in *Maple*

1. Find the three different cube roots of $1+i$. Express them in standard $x + iy$ form. (You might be tempted to do this by just evaluating $(1+i)^{1/3}$, but that will only get you one cube root, not all three. Instead, use `solve` to get all three.)

2. Find the real and imaginary parts of $(4+i)^{(4-i)}$. (The result is quite messy. Just think how much labor this would take by hand?)

3. Find $\sin^{-1}(16)$ (that is, *arcsin 16*).[1] Express the result in standard $x + iy$ form.

   a. Once you've done this, add 2π to the result, and take the sine of that. Make sure that this result makes sense to you. (What do you normally expect to happen when you take the inverse sine of a number, add 2π to it, and then take the sine?)

[1] **Note:** Recall that using only real numbers, you cannot take the inverse sine of any number greater than one, but in complex numbers you can.