Physics 350 Problem Set 9 (Spring Semester 2009)
Due **Fri, Apr. 17** at 4:30PM

1. (SET UP AND DO THE INTEGRAL BY HAND, but feel free to check your answer with Maple.) Find the Fourier transform of the function

$$f(x) = \begin{cases} x, & -1 < x < 1, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

   Recall from lecture that "find the Fourier transform" means "find the function $g(k)$ which is like the coefficients A_n and B_n in a Fourier series." The formula for $g(k)$ is

$$g(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx. \tag{2}$$

   Hint: Pay attention to the definition of $f(x)$ when setting up the limits of integration.

2. (SET UP AND DO THE INTEGRAL BY HAND, but feel free to check your answer with Maple.) Find the Fourier transform of the function $f(x) = exp(-|x|)$. Hint: You will need to break the integral over x from $-\infty \to \infty$ into the sum of two integrals, one integrating from $-\infty \to 0$ and the other from $0 \to \infty$.

3. (Use Maple for this one). The function $f(x) = e^{-x^2/\sigma^2}$, where σ is a positive constant, is called a Gaussian. Gaussians have the interesting mathematical property that the Fourier transform of a Gaussian is also a Gaussian,

$$g(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx = \sigma\sqrt{\pi}\ e^{-k^2\sigma^2/4}. \tag{3}$$

   Gaussians are interesting physically because they are an accurate description of particles in some potentials in quantum mechanics, and because they are central to error analysis in experimental physics.

   (a) Plot both $f(x)$ and $g(k)$ in Maple, preferably on the same graph, for several different values of σ. Your goal is to figure out from these graphs how the width of $f(x)$ is related to the width of $g(k)$.

(b) Interpret the relationship between width Δx of the peak $f(x)$ and the width Δk of the peak $g(k)$ in terms of the Heisenberg Uncertainty Principal (calling these "peaks" will make sense once you graph them). Note that in quantum mechanics the momentum p is related to k by $p = \hbar k$.