Physics 350 Problem Set 5 (Spring Semester 2009)
Due Thu., February 19 at 4:30PM

HINT FOR STUDENTS: You may use *Maple* (and are frankly encouraged) to do any of the problems on this problem set.

1. Boas 8.5.39 (**NOTE:** the graphs Boas asks for are for equations (5.30), (5.31) and (5.32) from Chapter 8). In part (a) also explain in words why the three solutions look different.

2. When discussing Taylor series we came up with an estimate of the error in the approximation we made if we truncated the series after n terms. In other words, we could estimate how big the error was in the Taylor series expansion of a function g if we cut off the function at $n = 10$ by looking at the size of the $n = 11$ term that we weren't including. If we define a similar error for the Fourier sine series, the magnitude of the error when the last term included is n is approximately $|B_{n+1}|$ (where B is the Fourier sine coefficient defined in lecture and in the last lab).

   For the function in part 2.b of Lab 6 ($f(x) = x - L/2$), what is this error for $n = 15$? Check how good this estimate of the error is by using Maple to calculate the average error, which we will define to be the average of $(f(x) - F_n(x))^2$ over the interval from 0 to L. In this expression $F_n(x)$ is the Fourier sine series of the function $f(x)$ including all terms *up to and including n*.

3. A function $f(x)$ is called symmetric (or even) if $f(-x) = f(x)$. A function $g(x)$ is called antisymmetric (or odd) if $g(-x) = -g(x)$. Any function $h(x)$ can be written as the sum of an antisymmetric part h_a and a symmetric part h_s. In other words, we can always write $h(x) = h_a(x) + h_s(x)$ even if h is neither even nor odd.

   (a) What is $h(-x)$ in terms of $h_a(x)$ and $h_s(x)$?

   (b) Show that $h_s = (h(x)+h(-x))/2$ and that $h_a = (h(x)-h(-x))/2$.

   (c) Find the symmetric and the anti-symmetric parts of the function $h(x) = (x^3 + x^2)\sin(x)$.

4. A common way of rewriting the Fourier series is to put the center of the interval at the origin. For the examples we did in lab this corresponds to making a change of variables $\tilde{x} = x - L/2$. For the each of the four functions in 3.a in Lab 6 rewrite the function in terms of $\tilde{x}$ and classify each function as either symmetric, antisymmetric, or neither.