Physics 350 Problem Set 4 (Spring Semester 2009)
Due Thu., February 12 at 4:30PM

In all of these questions be sure to **include an explanation for your answer**. No explanation, no credit.

1. (a) Solve the differential equation below *by hand.*

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 6x = 0. \tag{1}$$

   Your solution will have two constants in it that would be determined by initial conditions if you were given initial conditions.

   (b) Find the constants in your solution if $x(0) = 2$ and $v(0) = 0$.

2. Solve the differential equation below *by hand.*

$$9\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + x = 0, \tag{2}$$

   with initial conidtions $x(0) = 12$ and $v(0) = 5$.

3. The equation of motion for a damped driven oscillator is

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_N^2 x = f\cos(\omega_D t). \tag{3}$$

   (a) Find the particular solution to this equation *by hand.* That is, write the solution in the form

$$x = Ae^{i\omega_D t} \tag{4}$$

   and solve for A.

   (b) Show that the constant A in polar form is

$$A = \frac{f}{\sqrt{(\omega_N^2 - \omega_D^2)^2 + 4\beta^2\omega_D^2}}e^{i\delta}, \tag{5}$$

   where $\delta = \tan^{-1}(-2\beta\omega_D/(\omega_N^2 - \omega_D^2))$. Use Maple or do by hand as you see fit.

(c) Use the formula for A in the previous part to explain why the driven oscillator you looked at in lab this week was sensitive to the frequency ω_D. In particular, explain why the phase shift changed when the driving frequency is close to the natural frequency, and why the amplitude is much larger for some driving frequencies than others. Note: To make this match what we did in lab exactly, you would set $\omega_N = 2\pi$, $\beta = B/2$, and $f = 4\pi^2 f_{lab}/\omega_N^2$.

4. Starting with your lab 4 Maple worksheet, change the right hand side of the differential equation in problem 2 by adding a second driving term, $4\pi^2 f/\omega_N^2 \cos(1.5\omega_d T\eta)$. Plot the solution for three cases: $\omega_d \ll 2\pi$, $\omega_d \approx 2\pi$, and $\omega_d \gg 2\pi$.

5. The differential equation describing current flow in a series RLC circuit is of the same form as that of a damped oscillator. In other words the current satisfies the equation

$$B\frac{d^2 I}{dt^2} + D\frac{dI}{dt} + EI = 0. \tag{6}$$

find the constants B, D, and E in terms of R, L, and C. Recall that the voltage drop across each of these circuit elements is $V_{\text{resist}} = IR$, $V_{\text{cap}} = Q/C$, and $V_{\text{induct}} = -L(dI/dt)$, and that current is related to charge by $I = dQ/dt$.

6. Some more practice working with complex numbers. For each complex number z below find $|z|^2$ and put the number in polar form, if it is given in rectangular, and rectangular if it is given in polar.

(a) $\frac{i}{\sqrt{3}}$ (by hand)

(b) $4e^{i\pi/4}$ (by hand)

(c) $\frac{2+i}{1-i}$ (put in polar form and find $|z|^2$, do by hand or by Maple)