1. (a) The key here is to start by using the relationship $\delta(ay) = \frac{1}{|a|}\delta(y)$. The delta function in the integral is then

$$\delta(8x+1) = \frac{1}{8}\delta(x+1/8), \tag{1}$$

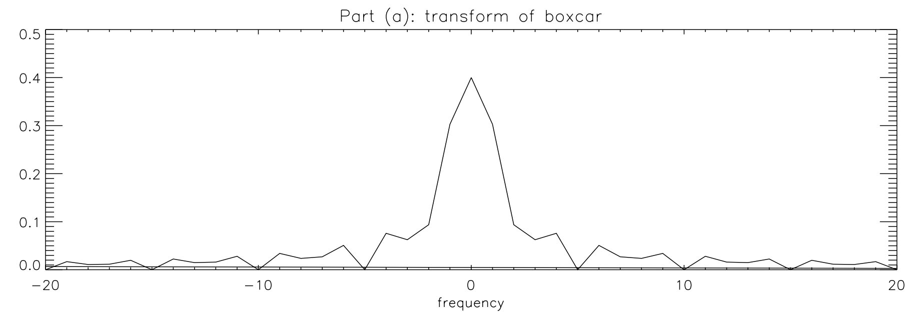
and the integral is

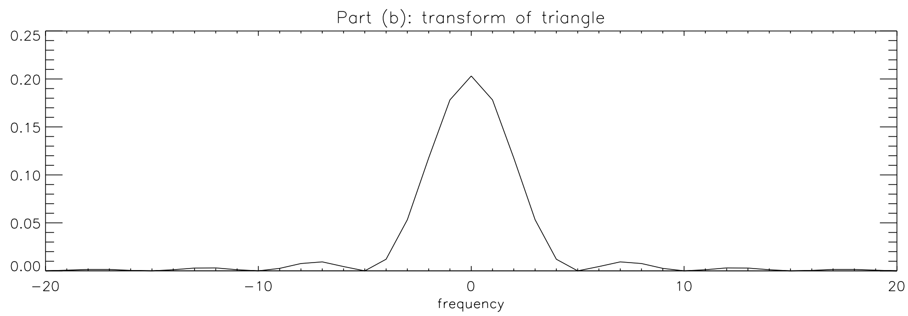
$$\int_{-\infty}^{\infty} x^2\delta(8x+1)dx = \frac{1}{8}\int_{-\infty}^{\infty} x^2\delta(x+1/8)dx = \frac{1}{8}\left(\frac{1}{8}\right)^2 = \frac{1}{512}. \tag{2}$$

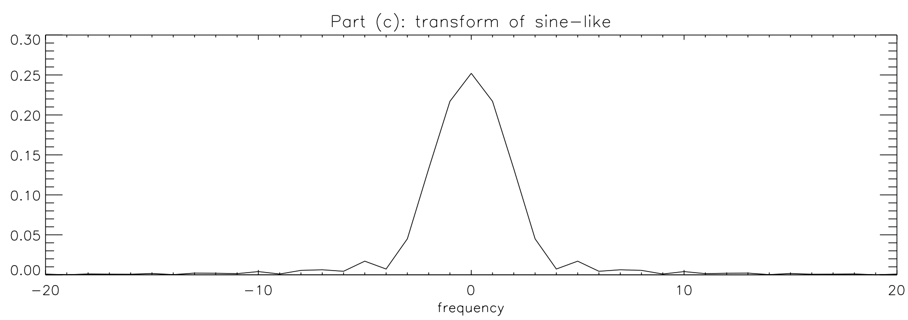
(b) The key to this one is that the delta function is zero everywhere except where its argument is zero. In this problem the argument of the delta function is $x+1$, which is zero when $x=-1$. The range of integration is from zero to ∞, and the delta function is zero everywhere in that range, so

$$\int_{-\infty}^{\infty} x\delta(x+1)dx = 0. \tag{3}$$

2. The graphs for parts (a), (b), and (c) are below.

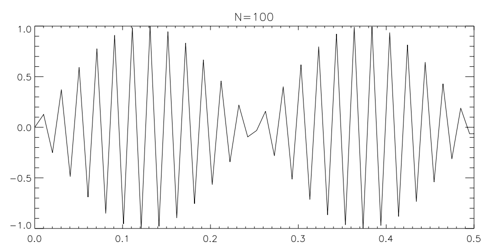


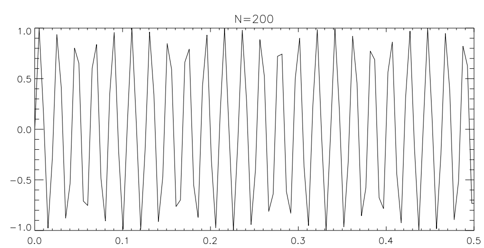


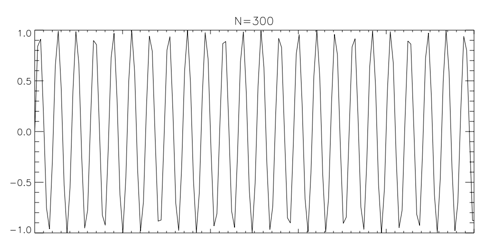


Part (d). Sharply varying functions, and discontinuities in particular, require many waves to reproduce. The boxcar requires a fairly large amplitude even at high frequencies because the sudden jump from 1 to 0 is hard to reproduce with nice, smooth sine and cosine functions. The triangle-shaped function is smoother, though there are sharp changes in the slope. If you compare it and the transform of the sine-like function between frequencies 10 and 15 you will note the transform of the triangle-like function is larger than that of the sine-like function. Of the three functions the sine-like function is the smoothest and its transform drops to zero at high frequency most rapidly.

3. The graph produced by the code in the assignment is below. Although it looks like the sum of sine waves (or some other beat-producing combination) the odd appearance is entirely due to the poor sampling in our graph. We are using 100 points, with a spacing of 1/99=0.0101 between them, to try to reproduce a curve whose wavelength is $2\pi/(95\pi) = 2/95 \approx 0.021$. With barely two samples per wavelength we cannot reproduce the original function. Put differently, the Nyquist frequency is $\alpha_N = (N/2)\alpha_f = 50\pi$, and the sine wave we are given is frequency 95π. Increasing the number of points to 200, the middle graph below, increases the Nyquist frequency to 100π, and the graph has improved quite a bit, though the amplitude still looks a bit off. Increasing N to 300 does even better.







4. An example of the IDL code to decode the message, from two files called `craig_I.png` and `craig_E.png` is below:

```
I = read_png('craig_I.png')
E = read_png('craig_E.png')

I_t = fft(I)
E_t = fft(E)

M_t = (E_t/I_t-1)/0.1
M = fft(M_t, /inverse)

window,0,xsi=600,ysi=600,title='Secret Message'
tvscl,M

end
```