Physics 350 Problem Set 11 (Spring Semester 2009)
Due **Thu, May 7** at 4:30PM

1. (DO THESE BY HAND, 5 points each.) Evaluate each of the integrals below.

   (a) $\int_{-\infty}^{\infty} x^2 \delta(8x+1)dx$ Note: the answer is *not* 1/64. Look at the solutions to HW10.

   (b) $\int_{0}^{\infty} x\delta(x+1)dx$ Note: The answer is *not* -1.

2. (DO IN IDL) For each of the functions below graph the function (over the range 0-20, using 100 points), find the Fourier transform and make a plot of the absolute value of the transform; turn in only the plots of the transform, and answer part (d) below. NOTE: The IDL code defining each of these functions is available on the course website.

   (a) A "boxcar" function,

   $$f(x) = \begin{cases} 1, & 6 < x < 14, \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

   (b) A "triangle" function,

   $$f(x) = \begin{cases} (x-6), & 6 < x < 10, \\ (14-x) & 10 \geq x < 14, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

   (c) A "sine" function set to zero outside a limited region,

   $$f(x) = \begin{cases} \sin(\pi x/8), & 6 < x < 14, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

   (d) The transforms of the functions above have different amounts of "ringing" (or oscillation) in them. Explain, in terms of the features of the functions $f(x)$, why the transforms have different amounts of ringing.

3. Run the code below in IDL; it is supposed to graph the function $\sin(95\pi x)$ over the range $0 \leq x \leq 1$.

```
N=100
x=findgen(N)/(N-1) ; make numbers from 0 to 1
plot,x,sin(95*!pi*x)
```

   (a) Does it actually graph a sine wave?

   (b) What needs to be changed to ensure the graph will actually be the desired sine wave?

4. You are to decode the secret message embedded in an image. The image containing the secret message is at the course website; each of you has a different image with a different secret message. You have received a copy of the image without the message by email. Use the two images to reconstruct the secret message. The image with the embedded message, called E, was generated from the image without the message, called I, and the message image, called M, by the following steps:

   - The Fourier transform of I and M were calculated.
   - The transforms were combined in this way to make the transform of E: $E_t = (1 + 0.1 * M_t) * I_t$ (the t indicates this is a relationship between the Fourier transforms of the images, in k-space).
   - The image E is the inverse transform of E_t. In other words, `E=fft(E_t, /inverse)` and `E_t=(1+0.1*M_t)*I_t`.

   Use this information, and the images E and M you have, to reconstruct the image M. Display it with `tvscl` and read your message. All of the images are 600x600...which doesn't really matter for solving the problem, but does suggest a size for the window you open. **The answer you will turn in for this question is the secret message that is embedded in your image.**