Physics 350 Problem Set 10 (Spring Semester 2009)
Due **Fri, Apr. 24** at 4:30PM

1. (DO THESE BE HAND, 5 points each.) Evaluate each of the integrals below.

   (a) $\int_{-\infty}^{\infty} \cos x\delta(x)dx$

   (b) $\int_{-\infty}^{\infty} x^2\delta(x-2)dx$

   (c) $\int_{3}^{10} x\delta(x-1)dx$

   (d) $\int_{0}^{\infty} x\delta(5x-1)dx$

2. Show that the Dirac delta function can be written as the limit of a Gaussian,

$$\delta(x) = \lim_{\sigma\to 0} g(x), \tag{1}$$

   where

$$g(x) = \frac{e^{-x^2/\sigma^2}}{\sigma\sqrt{\pi}}. \tag{2}$$

   How do you show this? The Delta function has a couple properties:

$$\int_{-\infty}^{\infty} \delta(x)dx = 1, \tag{3}$$

   and it is very narrowly peaked about the point $x = 0$. Use Maple to show that the function $g(x)$ satisfies the condition (3) as the delta function, make a plot that shows how $g(x)$ changes as σ gets smaller, and use that plot to argue that as $\sigma \to 0$ the function $g(x)$ gets closer and closer to an infinitely high but infinitely narrow peak.

3. Show that the Inverse Fourier Transform as given in class really is the inverse of the Fourier transform. From class

$$g(\alpha) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-i\alpha x}dx, \qquad \text{Fourier transform,} \tag{4}$$

$$f(x) = \int_{-\infty}^{\infty} g(\alpha)e^{i\alpha x}d\alpha, \qquad \text{Inverse Fourier transform.} \tag{5}$$

   What do I mean by showing that the inverse really is an inverse? Take the expression for $g(\alpha)$ given in (4) and put it into the inverse fourier transform (5). If the right hand side reduces down to $f(x)$ after simplifying, the inverse transform really does "undo" the transform. You may find useful the result from class that

$$\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\alpha x}d\alpha \qquad \Longrightarrow \qquad \int_{-\infty}^{\infty} e^{-i\alpha x}d\alpha = 2\pi\delta(x). \tag{6}$$