Physics 350 Mid-Term Exam Study Guide

Spring Semester 2009

The Physics 350 mid-term is scheduled for 3 pm – 5 pm on Wednesday, March 11. It will be given in Hagen 317 (our lab room). Unfortunately, we will **NOT** use *Maple* during the exam (you may now shed a tear).

- Exam is closed book and closed note.
- Critical equations will be provided.

Attached to this short topics list is a copy of the Spring 2007 Physics 350 Midterm. The topics were very similar to the ones on this exam. That exam proved a bit too long, but it gives you a flavor for the questions we might ask.

Here are some sample topics/concepts that might be covered on the Mid-Term. No specifics here, just the "form" of question that you might be called upon to solve.

Taylor Series

- Expand a straightforward function to a given order.
- Expand a physically meaningful function to the lowest non-zero order.

Complex Numbers

- Convert between Cartesian ($x+iy$) and Polar ($re^{i\theta}$) forms of complex numbers.
- Compute powers of complex numbers.
- Compute roots of complex numbers.

Differential Equations

- Know the standard solutions for
    - Linear differential equations with constant coefficients (both with and without repeated roots) and right hand size equal to zero. (e.g. - undamped and damped harmonic oscillators)
    - Linear differential equations with constant coefficients with non-zero right hand side. (e.g. - driven harmonic oscillators, both damped and undamped)
- What are the characteristic and particular solutions to a driven harmonic oscillator and what do they mean?
- Why would Fourier series be important for solving some forms of the driven harmonic oscillator problem?

Fourier Series

- Which functions contain only Fourier sine series terms? Which functions contain only Fourier cosine series terms?
- Calculate series expansion of a trigonometric function like $\sin^3\left(\frac{2\pi x}{L}\right)$.
- Setup but don't evaluate expressions for A_n and B_n given some function like xe^{-x}.

Inner Products

- Find $\langle f|g\rangle$, for example for $f=x$ and $g=x^2$ with $\langle f|g\rangle \equiv \int_{-1}^{1} fg\,dx$ (the bounds on this integral come from the allowed range of x for Legendre polynomials, your bounds may vary, why?).
- Show that a pair of functions is orthogonal.
- Derive an expression for the coefficients of a series built with some orthonormal functional basis.

Matrices and Linear Transforms

- Find the product of two matrices A and B.
- Apply a rotational matrix to a vector to find the rotated vector.
- Show that a rotation of 90° about the x axis followed by a rotation of 90° about the z axis will not have the same results as a rotation of 90° about the z axis followed by a rotation of 90° about the x axis.

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Each part of each problem is worth five (5) points unless otherwise noted. There are a total of 75 points possible on this exam. Problems with only one part are worth five points unless otherwise noted. Please box your final answers.

1. The velocity of an object dropped from rest that experiences linear air resistance is $v(t) = g(e^{-bt} - 1)/b$, where g is the acceleration due to gravity and b is a positive constant with units sec^{-1}.
   (a) Show that at early times $v(t) \approx -gt$ as you would expect without air resistance.
   (b) Since this is a course in computational physics, what do you think you gain computationally by using $v(t) \approx -gt$ instead of $v(t) = g(e^{-bt} - 1)/b$ at early times?
   (c) Explain how you could use a computer program (like *Maple*) to check at what times the approximation $v(t) \approx -gt$ fails by an unacceptable amount. We are not asking you to write the code, we are asking to explain what computations you would ask the computer to make.

2. Write the number or numbers z in each part below in both Cartesian $(x + iy)$ form and in polar $(re^{i\theta})$ form. *Show your work.*
   (a) $z = (1 + i)^2$
   (b) $z = \frac{5\sqrt{2}}{1-i}$
   (c) $z^4 = i$

3. **(10 points)** Recall in class when we showed that using Euler's equation it was possible to prove the two double angle formulae? Use the same technique to prove the two half-angle formulas:

$$\cos(2\theta) = 2\cos^2\theta - 1 \quad (1)$$

$$\sin(2\theta) = 2\sin\theta\cos\theta \quad (2)$$

   Hint: Start with the Euler equation, $e^{i\theta} = \cos\theta + i\sin\theta$ and the fact that $e^{i(2\theta)} = (e^{i\theta})^2$. Solve for this complex equation to verify the two half-angle formulas.

4. Consider the function $g(x) = \cos^2\left(\frac{\pi x}{L}\right)$.

   (a) Is the Fourier series for $g(x)$ a sine series, a cosine series, or both? *Explain your reasoning. Hint: No computations may be necessary.*

   (b) Use the half-angle formula Eq. (1) to rewrite $g(x)$ in terms cosines without powers. Explain why this is the Fourier series expansion for $g(x)$, and check whether it agrees with your answer in part a.

5. Verify that the functions $P_1(x) = x$ and $P_3(x) = \frac{1}{2}(5x^3 - 3x)$ are orthogonal on the interval from -1 to 1.

6. Find the product of the matrices below.

   (a) $\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 3 & -2 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

7. Find the components of the vector

$$\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{3}$$

   after it has been rotated by an angle of $3\pi/4$ about the z-axis. *Hint:* Start with a sketch of the initial vector and figure out where it should end up so that you can check your work.