Physics 350 Final Exam Study Guide

Spring Semester 2009

The Physics 350 final exam is scheduled for 9 am – 11:30 am on Friday, May 11. It will be given in Hagen 316 (our lab room). Unfortunately, since the *IDLs* license will NOT have expired by then, we WILL be able to use *IDL* during the exam (you may now shed a tear).

- Exam is closed book and closed note.
- Critical equations will be provided.
- You are certainly allowed to query help in *IDL* if necessary or even ask Dr. Cabanela or Dr. Craig for help with a *IDL* command used for a problem.

Attached to this short topics list is a copy of the questions from the Spring 2007 Physics 350 Final Exam that covered the same material we covered this semester. One problem covered contour integrals and we are not including that beastie here. **KEEP IN MIND, IN THIS FINAL, WE MAY WELL INCLUDE A FOURIER TRANSFORM PROBLEM IN PLACE OF THE DROPPED CONTOUR INTEGRAL PROBLEM.** Other than that problem, the exam worked out well, even with the IDL problem. I am not uploading the files related to that IDL problem (can't find them right now), but it gives you a flavor of what we were asking about.

Here are some sample questions that might be covered on the Final Exam. No specifics here, just the "form" of question that you might be called upon to solve.

Returning from our Mid-Term for a command performance during the Final Exam:

Differential Equations

- Know the standard solutions for
  - Linear differential equations with constant coefficients (both with and without repeated roots) and right hand size equal to zero. (e.g. - undamped and damped harmonic oscillators)
  - Linear differential equations with constant coefficients with non-zero right hand side. (e.g. - driven harmonic oscillators, both damped and undamped)
- What are the characteristic and particular solutions to a driven harmonic oscillator and what do they mean?
- Why would Fourier series be important for solving some forms of the driven harmonic oscillator problem?

All new for the Final Exam:

Matrices and Linear Transforms [New since the Mid-Term]

- Know how *IDL* represents matrices and know how to handle basic matrix multiplication in *IDL*.

Fourier Transforms

- Know what the Fourier transform does when working with a function of position, *f(x)*, or time, *g(t)*. What are the 'units' of the Fourier transform of *f(x)* or *g(t)*?
- If *f(x)* is a "narrow" function (having only a small range of values of *x* where *f(x)* is non-zero), what do you expect the rough shape of the Fourier transform of *f(x)* to be and what does it mean?
- For discrete Fourier Transforms
    - Be able to determine the Nyquist frequency (and know why it is important).
    - Be able to determine the Fundamental frequency (and know why it is important).
- Describe the effect of removing low or high spatial frequency components from an image. Or stated "inversely," what do the low and high spatial frequency components contribute to an image.
- Know how to implement a (discrete) Fourier Transform of an arbitrary function or data sample in *IDL*.

Convolution

- Know what a convolution is and how Fourier transforms can ease the computation of a convolution.
- Know how *IDL* can be used to perform a convolution, specifically how you implement the technique we refer to in the previous bullet point.

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Spring Semester 2007

Each part of each problem is worth five (5) points unless otherwise noted. Problems with only one part are worth five points unless otherwise noted. Please $\boxed{\text{box}}$ your final answers.

Pencil and paper problems—no computer or calculator allowed

1. Imagine sampling the output of a microphone once every millisecond for two seconds. What are the highest and the lowest frequencies you can accurately represent? Explain how you arrived at your answer, or clearly show the work leading up to the answer.

2. Find **all** the solutions for z in the following equations and express those solutions in polar ($re^{i\theta}$) form.

   (a) $z^3 = \frac{1+i}{\sqrt{2}}$.

   (b) $z^{8/3} = -1$.

3. The number of electrons of energy E as a function of temperature in a metal is proportional to

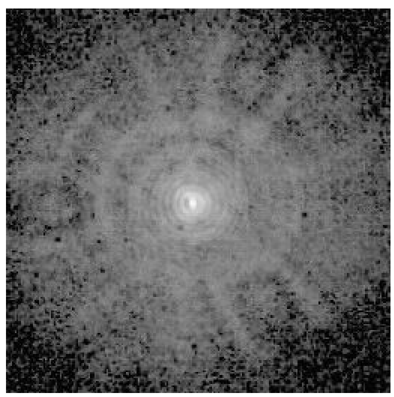
$$n(E) = \frac{1}{1 + e^{(E-E_f)/(kT)}}, \tag{1}$$

   where k is the Boltzmann constant, T is the temperature of the metal (which we will treat as a constant), and E_f is a constant called the Fermi energy (the Fermi energy is the energy of the highest filled state when the metal is at absolute zero). Expand $n(E)$ about the point $E = E_f$ to the lowest non-trivial order. Non-trivial means the lowest order that actually contains an E.

4. CONTOUR INTEGRATION PROBLEM REMOVED! Expect this to be replaced by something related to Fourier Transforms.

Computer problem — You may use a computer (and the internet) for this section.

5. **(20 points)** In 1990, the Hubble Space Telescope (HST) was launched to much fanfare. The launch had been delayed by the explosion of the Space Shuttle Challenger and the overruns had cost NASA over \$1 Billion. So it was a major disappointment when the first images came back from the Faint Object Camera (FOCAS) came back and they were blurry. It was discovered the main mirror was 2 μm out of shape, which resulted in the telescope not focusing light as well as it should. As such stars, instead of appearing almost pointlike in an image, appeared as large diffuse objects. How a pointlike star appears in a telescope is called the telescope's point spread function (PSF).



The Point Spread Function (PSF) of the HST FOCAS Camera measured by pointing the HST at a isolated bright star.

Every image taken by a telescope is effectively convolved with the PSF. In the last lab, you saw how increasing the radius of the function you were convolving with made the resulting image more blurred. The large PSF of the FOCAS camera effectively blurred every image taken with the camera. NASA scientists eventually developed software to allow their observers "deconvolve" the PSF from the blurred images taken by the FOCAS camera. As a result, a lot of reasonably sharp images were taken from the FOCAS camera before the optics in the telescope were corrected with the first servicing mission in 1993.

Recall that if we convolve an image described by function $f(x)$ by a

PSF described by a function $p(x)$, the observed image $i(x)$ is given by

$$i(x) = (f * p)(x) \equiv \int f(x')p(x - x')\ dx \tag{2}$$

and can be solved for more efficiently by working with the Fourier Transforms of $f(x)$ and $p(x)$ such that

$$FT[i(x)] = FT[(f * p)(x)] = FT[f(x)] \times FT[p(x)] \tag{3}$$

In deconvolution, we try to recover the original non-convolved image, $f(x)$, from the observed image $i(x)$ and PSF $p(x)$ by noting that:

$$FT[f(x)] = \frac{FT[i(x)]}{FT[p(x)]} \tag{4}$$

Your goal for this portion of the exam is to write a short *IDL* routine to take a blurry image that has been convolved with a known PSF and deconvolve the PSF from the blurred image to recover the original unconvolved image. A blurry image, PSF image, and some starter *IDL* code are provided for download in a ZIP package at `http://phys350.cabanela.com/media/final_deconvolution.zip`. Download this file to your `~/idl` directory and unzip the package there. The following files will be placed in your directory:

- `deconvolve.pro` is an *IDL* procedure file with a few lines in it for reading the blurred image data, which is stored in a proprietary *IDL* format. This is the file you will edit and printout.
- `blurred_image.dat` is an 1024×1024 image of an image that has been convolved with the HST FOCAS PSF. The image is in a proprietary *IDL* format.
- `FOCAS_PSF.jpg` is an 1024×1024 image of the HST FOCAS PSF shifted such that the center of the image is at the origin.

Modify `deconvolve.pro` to load the PSF image and then recover the original sharp image by performing a deconvolution on the blurred image. For full credit you must provide

(a) A description of the blurred image.

(b) A description of the PSF image

(c) A description of the final image.

(d) A printout of your final `deconvolve.pro` file. We expect the file to be well commmented with a clear description of your logic for every part of the code. If printing from Xemacs be sure to select "Pretty print buffer" rather than "Print buffer."