Physics 322 Problem Set #9
(Let's get more real ... in 3-D)

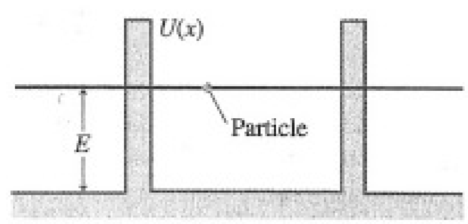
Due Friday, April 17 at 4:00 pm

ASSUMED READING: Before starting this homework, you should have read Sections 1 to 4 of Chapter 7 of Harris' *Modern Physics*.

SCORING: There are 60 points possible on this Problem Set (not including extra credit). Scoring per problem is indicated.

HINT: The problem I consider to be the algebraically and computationally messy one, Problem 6 (Harris 7.26), can be done without reading the chapter, you might want to start it early and get it out of the way.

1. **[Harris 6.1] (5 points)** Could the situation depicted in the diagram to the right represent a particle in a bound state? Explain.



2. **[Harris 7.1 modified] (5 points)** What is a quantum number, and how does it arise? In answering this question, clarify what it physically represents.

3. **[Harris 7.3 tweaked] (10 points)** Consider a 2D infinite well whose sides are of unequal length with the "horizontal" (x-axis) length being longer than the "vertical" (y-axis) length.
   - **a.** Sketch the probability density – as density of shading – for the ground state (in other words, darker shades in a sketch mean higher probability density).
   - **b.** There are two likely choices for the next lowest energy. Sketch the probability density and explain how you know this must be the next lowest energy. (Focus on the qualitative idea, avoiding unnecessary reference to calculations).

4. **[Harris 7.21] (10 points)** An electron is trapped in a (cubical) quantum dot, in which it is confined to a very small region in all three dimensions. If the lowest-energy transition is to produce a photon of 450 nm wavelength, what should be the width of the well (assumed cubic)? **WARNING:** Be sure to justify why this transition must be the lowest-energy transition.

5. **[Harris 7.22 tweaked] (10 points)** Consider a cubic 3D infinite well.
   - **a.** How many different wave functions have the same energy as the one for which $(n_x, n_y, n_z) = (5, 1, 1)$? **HINT:** Table 7.1 might help speed things up.
   - **b.** Into how many different energy levels would this level split if the length of one side (say the z-axis) were increased by 5%?
   - **c.** Make a scale diagram, similar to Figure 7.3, illustrating the energy splitting of the previous degenerate wave functions.
   - **d.** Is there any degeneracy left? If so, how might it be "destroyed"?

6. **[Harris 7.26 tweaked] (10 points)** Classically, an orbiting charged *particle* radiates electromagnetic energy, and for an electron in atomic dimensions, it would lead to the collapse in considerably less than the wink of an eye.
   - **a.** By equating the centripetal and Coulomb forces, show that for a classical charge $-e$ of mass m held in circular orbit by its attraction to a fixed charge $+e$, the following relationship holds:

     $$\omega = \frac{er^{-3/2}}{\sqrt{4\pi\epsilon_0 m}}$$

   - **b.** Electromagnetism tells us that a charge whose acceleration is a radiates power $P = \frac{e^2a^2}{6\epsilon_0 c^3}$. Show that this can also be expressed in terms of the orbit radius, as $P = \frac{e^6}{96\pi^2\epsilon_0^3 m^2 c^3 r^4}$ *[There is a typo in this equation in the book]*. Then calculate the energy lost per orbit in terms of r by multiplying this power by the period $T=2\pi/\omega$ and using the formula from part (a) to eliminate ω.

c. In such a classical orbit, the total mechanical energy is one half the potential energy (see Section 4.4) or $E_{orbit} = -\frac{e^2}{8\pi\epsilon_0 r}$. Calculate the change in energy per change in r: $\frac{dE_{orbit}}{dr}$ and use this to relate a change in radius dr to a change in energy dE_{orbit}. From this and the energy lost per orbit from part (b), determine the change in r per orbit and evaluate it for a typical orbit radius of 10^{-10} m. Would the electron's radius change much in a single orbit?

d. Argue that dividing dE_{orbit}/dr by P and multiplying by dr gives the time required for r to change by dr. Then, sum these times for all radii from $r_{initial}$ to a final radius of 0 (**HINT:** Does the electron need to gain energy or lose energy to drop in radius, make the appropriate assumption in the value of dE_{orbit}/dr). Evaluate your result for $r_{initial} = 10^{-10}$ m. (one limitation of this estimate is that the electron would eventually be moving relativistically). What does this imply about the classical model of the atom electrons orbiting a positively charged nucleus?