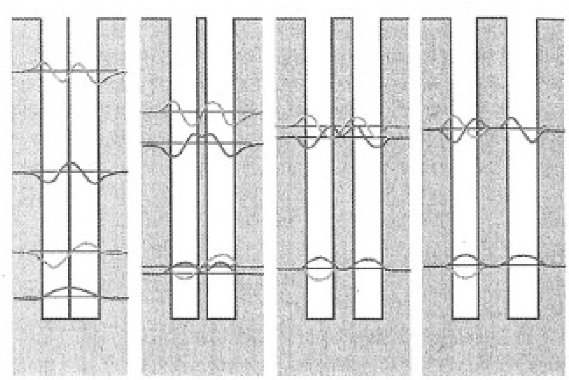
Physics 322 Problem Set #7
(Tunneling toward Spring Break)

Due Friday, March 13 at 4:00 pm

ASSUMED READING: Before starting this homework, you should read Chapter 5.6 through 5.8 of Harris' *Modern Physics*.

SCORING: There are 35 points possible on this Problem Set (not including extra credit). Scoring per problem is indicated.

1. **[Harris 5.12 tweaked] (5 points)** Simple models are very useful. Consider the twin finite wells shown in the figure, at first with a tiny separation, then with increasingly distant separations. In all cases, the four lowest allowed wave functions are plotted on axes proportional to their energies. We see that they pass through the classically forbidden region between the wells, and we also see a trend. When the wells are very close, the four functions and energies are what we might expect of a single finite well, but as they move apart, pairs of functions converge to intermediate energies. **HINT:** You can visualize these levels by going to the electronic handouts page on our website and following the link to the "Quantum Bound States" PhET demo. Run the demo. Switch to the "Two Wells (Molecular Bonding)" tab. Click on "wave function" (instead of probability density). Now you can simulate everything in the above image.



   a. The energies of the second and fourth states decrease. Based on changing wavelength alone, argue that this is reasonable. **HINT:** What happens to the wavelength of the wave function for the even-numbered energy levels as the gap between the two potential well increases and why?
   b. The energies of the first and third states *increase*. Why? (**Hint:** Study how the behavior required in the classically forbidden region affects these two relative to the others.)
   c. The distant-wells case might represent two distant atoms. If each atom had one electron, what advantage is there in bringing the atoms closer to form a molecule? (**Note:** Two electrons can have the same wave function.)

2. **[Harris 5.47 Modified] (10 points)** Consider the delta well potential energy:

$$U(x) = \begin{cases} 0 & x \neq 0 \\ -\infty & x = 0 \end{cases}$$

Although not completely realistic, this potential energy is often a convenient approximation to a *very* strong, *very* narrow attractive potential energy well. It has only one allowed bound-state wave function, and because the top of the well is defined as $U=0$, the corresponding bound-state energy is negative. Call its value $-E_0$.

   a. Applying the usual arguments and required continuity equations (need it to be smooth?), show that the wave function is given by

   $$\psi(x) = \left(\frac{2mE_0}{\hbar^2}\right)^{1/4} e^{-\left(\sqrt{2mE_0}/\hbar\right)|x|}.$$

   BIG HINT: There are only two regions in this problem, $x>0$ and $x<0$, whose solutions you have to get to work. The first derivative of the wave function, $d\psi/dx$, will be discontinuous here. This is acceptable in this case because this is a limiting case of an infinitesimally wide potential well, something that doesn't exist in reality, but is a useful approximation in some cases.

   b. Sketch $\psi(x)$, $U(x)$, and $-E_0$ on the same diagram. Does this wave function exhibit the expected behavior in the classical forbidden region?

3. **[Harris 5.55] (10 points)** Classically, if a particle is not observed, the probability per unit length of finding it in a box is a constant $1/L$ along the entire length of the box. With this, show that the classical expectation value of the position is $L/2$, that the expectation value of the square of the position is $\frac{1}{3}L^2$, and that the uncertainty in position is $\frac{L}{\sqrt{12}}$.

4. **[Harris 5.56] (10 points)** Show that the uncertainty in a particle's position in an infinite well in the general case of arbitrary n is given by

$$L\sqrt{\frac{1}{12} - \frac{1}{2n^2\pi^2}}.$$

Discuss the dependence. In what circumstances does it agree with the classical uncertainty of $\frac{L}{\sqrt{12}}$ discussed in the previous problem?