

## Physics 322 Problem Set #4 (Waves of Matter)

Due Friday, February 13 at 4:00 pm

**ASSUMED READING:** Before starting this homework, you should read Chapter 4.1 through 4.4 of Harris' *Modern Physics*.

**SCORING:** There are 60 points possible on this Problem Set. Scoring per problem is indicated.

1. **[Harris 4.2 modified] (5 points)** Generally speaking, why is the wave nature of matter so difficult to observe in everyday life?
2. **[Harris 4.4 modified] (5 points)** All other things being equal, which would be more likely to exhibit its wave nature – a proton or an electron – and why? What physical property could you adjust in the less wave-like particle so as to make it as wavelike as the other? Clearly state how *unequal* you would have to make that physical property between the electron and the proton to make them behave equally wave-like.
3. **[Harris 4.8] (5 points)** When we refer to a “bound” particle, we usually mean one for which there is no probability of finding it outside some finite confines. Could a bound particle be perfectly dead stationary, meaning a well-defined velocity of zero? Why or why not?
4. **[Harris 4.11 tweaked] (5 points)** A beam of electrons strikes a barrier with two narrow but equal width slits. A screen is located beyond the barrier, and electrons are detected as they strike the screen. The “center” of the screen is the point equidistant from the slits. When either slit alone is open, 10 electrons arrive per second in a very small region at the center of the screen. When both slits are open, how many electrons will arrive per second in the same region at the center of the screen? *Clearly explain your reasoning!*

5. **[Harris 4.24 tweaked] (10 points)** The Moon orbits Earth at a radius of  $3.84 \times 10^8$  m. To do so as a classical particle, its wavelength should be small. But small relative to what? Being a rough measure of the region where it is confined, the orbit radius is certainly a relevant dimension against which to compare the wavelength. Compare the two. Does the Moon indeed orbit as a classical particle? (**HINT:** You can estimate the Moon's velocity assuming it is in a circular orbit where the centripetal force is provided by gravity between  $m_{Earth} = 5.98 \times 10^{24}$  kg and  $m_{Moon} = 7.35 \times 10^{22}$  kg).
6. **[Harris 4.25 reworded] (10 points)** In the hydrogen atom, the electron's orbit, not necessarily circular, extends to a distance of about an ångström ( $1 \text{Å} = 0.1 \text{ nm}$ ) from the proton. If it is to move about as a compact classical particle in the region where it is confined, the electron's wavelength had better *always* be much smaller than an ångström. Here we investigate how large the electron's wavelength might be.
- If orbiting as a particle, its speed at  $1 \text{Å}$  could be no faster than that for *circular* orbit at that radius if the orbit is to extend no further than  $1 \text{Å}$  from the proton. *Why?*
  - Assuming it is an electrostatic force that keeps the electron in a circular orbit, compute the corresponding wavelength for the electron and compare it to  $1 \text{Å}$ . Can the atom be treated classically?
7. **[Harris 4.34 tweak]** Because we have found no way to formulate quantum mechanics based on a single real wave function, we have a choice to make. In Section 4.3, it is said that our choice of using complex numbers is a convenient one. Show that the free-particle Schrödinger equation (Harris 4-8)
- $$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$
- is equivalent to two *real* functions, as follows:
- $$\frac{\hbar^2}{2m} \frac{\partial^2 \Psi_1(x,t)}{\partial x^2} = \hbar \frac{\partial \Psi_2(x,t)}{\partial t} \quad \text{and}$$
- $$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi_2(x,t)}{\partial x^2} = \hbar \frac{\partial \Psi_1(x,t)}{\partial t}$$
- where  $\Psi(x,t)$  is by definition  $\Psi_1(x,t) + i\Psi_2(x,t)$ . How is the complex approach chosen in Section 4.3 more convenient than the alternative posed here?
8. **[Harris 4.42] (10 points)** A mosquito of mass 0.15 mg is found to be flying at a speed of 50 cm/s within an uncertainty of 0.5 mm/s.
- How precisely may its position be known?
  - Does this inherent uncertainty present any hinderance to the application of classical mechanics?