Physics 322 Problem Set #10
(Keeping it real ... in 3-D)

Due Friday, May 1 at 4:00 pm

ASSUMED READING: Before starting this homework, you should have read Sections 1 through 8 of Chapter 7 of Harris' *Modern Physics*.

SCORING: There are 65 points possible on this Problem Set (not including any extra credit). Scoring per problem is indicated.

1. **[Harris 7.28] (10 points)** Show that of hydrogen's spectral series – Lyman, Balmer, Paschen, and so on – only the four Balmer limes of Section 7.3 are in the visible range (400 - 700 nm). **NOTE ON YOUR APPROACH:** To show this, you must show that (i) other hydrogen spectral line series can't produce visible spectral lines and (ii) that only four visible wavelength Balmer spectral lines exist. This is more an argument from logic than a mathematical proof, but some math will be necessary to make the point.

2. **(5 points extra credit)** In astronomy there is a phenomenon seen in the spectra of many stars. When you examine the spectrum, it appear to be a blackbody spectrum with some absorption lines corresponding to spectral lines of elements in the star's atmosphere. But there is a sharp drop in intensity at wavelengths less than 912Å (in the rest frame of the star), which corresponds to a photon energy of 13.6eV. This intensity drop given the name the "Lyman break" and as the name implies it it associated with the Lyman series of spectral lines corresponding to transitions to/from the ground state of hydrogen. Assume the blackbody spectrum is produced in the interior of a star and then streams out through the outer atmosphere of the star. The outer atmospheres of most stars consist largely of hydrogen in this ground state (its a bit of a stretch for the very hot stars, but its correct for 90% of them). Explain the presence of the Lyman break in stellar spectra. How would you confirm the presence of the hydrogen necessary to cause the Lyman break in the outer atmosphere of the star? **HINT:** You don't need to do a lot of math here, but you need to be very clear in your explanation (and some diagrams might help).

3. **[Harris 7.37 extended] (10 points)** An electron is in the $\ell = 3$ state of the hydrogen atom.
   a. What possible angles might the angular momentum vector make with the *z*- axis?
   b. What possible angles might the angular momentum vector make with the *x*- axis? *y*-axis? **NOTE:** To be clear, I am not suggesting you know the angle between the angular momentum vector and the *z*-axis while also knowing the other ones. I am suggesting if you were to measure along the *x*- or *y*- axis instead of the z- axis, what would you measure?

4. **[Harris 7.39 extended] (10 points)** In Section 7.5, $e^{im_\ell \phi}$ is presented as our preferred solution to the azimuthal equation, but there is a more general one that need not violate the smoothness condition, and that in fact covers not only complex exponentials, but also, with suitable redefinitions of multiplicative constants, sine and cosine
   $\Phi_{m_\ell}(\phi) = Ae^{+im_\ell \phi} + \mathrm{B}\,\mathrm{e}^{-im_\ell \phi}$
   a. Show that the complex square of this function is not, in general, independent of ϕ.
   b. What conditions must be met by *A* and/or *B* or the probability density to be rotationally symmetric – that is, independent of ϕ?
   c. The original version of this problem in Harris parenthetically notes "This highlights another reason, besides their being of well-defined L_z, why we like our preferred solutions." Why is rotational symmetry a desired property for a solution to the wave equation for a central force?

5. **[Harris 7.45 tweaked] (10 points)** An electron is in an *n*=4 state of the hydrogen atom.
   a. What is its energy (Verify equation 7-14 comes from equation 7-12 and you can use 7-14)?
   b. What properties besides energy are quantized, and what values might be found if these properties were to be measured?

6. **[Harris 7.48] (10 points)** Show that the normalization constant $\sqrt{15/32\pi}$ given in Table 7.3 for the angular parts of the $\ell = 2, m_\ell = \pm 2$ wave function $\Theta_{2,\pm 2}(\theta)\Phi_{\pm 2}(\phi) = \sqrt{\frac{15}{32\pi}} \sin^2\theta e^{\pm 2i\phi}$ is correct. **Hint:** You are working in spherical coordinates, so you need the normalization condition given in equation (7-37). <u>Justify</u> your use of this equation, don't just use it because I told you to. Do

"justify" your use, you must answer "why would this equation be at all appropriate" and "what does an equation need to do to be a normalization condition?" **Hint #2:** $\sin^5\theta$ can be written as $(1-\cos^2\theta)^2\sin\theta$, which if you expand, can make for an easier to tackle integral.

7. **[Harris 7.56 extended] (15 points)** For a hydrogen atom in the ground state, determine
   - **a.** the most probable location at which to find the electron and
   - **b.** the most probable radius at which to find the electron (yes, it's different than (a)).
   - **c.** Comment on the relationship between your answers in parts (a) and (b). Specifically why are these results different? Don't just cite the equations, explain in words a non-physics major [but maybe a math major] could understand. **HINT:** How does the area of a spherical shell vary with radius? Why does that matter?
   - **d.** Recall that the potential energy *U(r)* in the hydrogen atom is just the Coulomb potential

     $$U(r) = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}.$$

     If we consider that in the ground state of a hydrogen atom, the electron has energy:

     $$E_1 = -\frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2}$$

     Then the classical limit on the radius of the electron's orbit occurs when $E_1 - U(r) = KE = 0$, such that $E_1 = U(r)$, and thus we can compute the maximum classically allowed radius of a ground state electron:

     $$E_1 = U(r)$$

     $$-\frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2} = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

     $$r = \frac{e^2}{4\pi\epsilon_0}\frac{2(4\pi\epsilon_0)^2\hbar^2}{me^4}$$

     $$= \frac{2(4\pi\epsilon_0)\hbar^2}{me^2}$$

     $$\therefore \boxed{r = 2a_0}$$

     But, as I noted, this is the classical limit. What is the probability that a ground state electron will lie outside this limit? Comment on the magnitude of the probability. **NOTE:** The integral you will need to tackle is best tackled by repeated integration by parts.