Magnitudes: Astronomers' Method of Defining Brightness

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This handout is primarily designed to help students unfamiliar with the magnitude system with understanding not only its definition, but how it is used in astronomy. This document focuses on constructing definitions for the following terms:

- apparent magnitude (m)
- absolute magnitude (M)
- distance modulus ($m-M$)
- surface brightness (μ [mag/arcsec^2] or I [L_{solar}/pc^2])

The Greek astronomer Hipparchus (2nd Century BCE) is credited with the construction of one of the first catalogs of the stars in the night sky, recording the positions and brightnesses of about 850 stars. Since Hipparchus had no access to any device for measuring brightness other than his eyes, he classified stars into six categories of brightness, ranging from the brightest "1st magnitude" stars to the faintest visible "6th magnitude" stars. Today we still use a magnitude system to categorize the brightnesses of stars, although it has become a much more precisely defined system than it was 22 centuries ago.

The modern magnitude system can be traced to the publication in 1856 of "Magnitude Constants for Fifty-seven Minor Planets" by Norman Pogson (Pogson, N. 1856, MNRAS, **17**, 12). He noted, as had William Herschel, that the magnitude scale was roughly logarithmic[2]. That is while you may perceive one star to be only a few times brighter than another, the actual flux (energy per unit area per unit time hitting your eye) from the two stars may differ by several orders of magnitude. What the eye perceives as linear steps in brightness actually involves changes by multiplicative factors for energy entering the eye (per unit time per unit area of the eye).[3] Pogson also noted that first magnitude stars were about a hundred times brighter than sixth magnitude stars. Therefore, Pogson defined a *magnitude difference* of five to correspond to a *brightness ratio* of 100. So a one magnitude difference corresponds to the fifth root of 100 or ~2.5… mathematically the "law of magnitudes" is written

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[1] This handout is based in large part on documents placed online by the Astronomy department at Northwestern University at <*http://www.astro.northwestern.edu/labs/m100/mags.html*> and the University of Queensland <*http://www.physics.uq.edu.au/people/ross/phys2080/survey/mags.htm*>, although I have extensively edited the content to match the needs of Astronomy 410 students and added a discussion of surface brightness.

[2] The human eye is actually better fit as a power law detector than a logarithmic detector, but that discussion is for another day.

[3] This is fortunate; if the eye responded linearly instead of logarithmically to light intensity, you would be able to distinguish objects in bright sunlight, but would be nearly blind in the shade!

$$m_1 - m_2 = -2.5 \log_{10} \frac{F_1}{F_2} \qquad [1]$$

where F_1 and F_2 are two brightnesses (or in astronomical language, fluxes) and $m_1 - m_2$ is the corresponding magnitude difference. The inverse of this relation (you should show this) is just:

$$\frac{F_1}{F_2} = 10^{-0.4(m_1 - m_2)} \qquad [2]$$

To standardize the system, astronomers have chosen to make the star Vega the photometric standard with an assumed magnitude of zero in all bandpasses. Thus when astronomer tells you that the human eye can see stars down to 5th magnitude in light polluted skies, they are telling you they can see stars up to 1/100 the brightness of Vega [try out equation 2 and show this last statement to be true].

Table 1: apparent magnitudes for some astronomical objects

Object	**magnitude**
Sun	–26.8
Full Moon	–12.5
Venus (at its brightest)	–4.4
Jupiter	–2.2
Sirius (brightest star in sky, other than Sun)	–1.6
Vega	0.0
Betelgeuse	+0.4 (variable)
Polaris	+2.0 (variable)
Limit of human eye (in dark skies)	+6.5
Limit with 10x50 binoculars (in dark skies)	+10
Limit of the Palomar Sky Survey	+21

Although the magnitude system can be awkward, it is so deeply ingrained in astronomical literature (and databases) that it would be difficult to abandon the system now. Some of the "awkward" features of the magnitude system that merit mention include:

- Recall Hipparchus declared 1st magnitude stars to be the brightest and 6th magnitude the faintest. Pogson, conserved this aspect of Hipparchus' system by creating a magnitude system in which *brighter* objects have *smaller* magnitudes. As an example, look at table 1 above, with the magnitudes of various objects.
- The magnitude system is logarithmic, which means each step in magnitude is an increase in the *ratio* of brightness. In other words, a magnitude 1 star is 2.5 times fainter than Vega [magnitude 0, remember?], but a magnitude 3 star is $2.5^2 \sim 6.25$ fainter than Vega! Always use equation [2] if you get confused about what a magnitude difference corresponds to.
- The magnitude you observe can vary radically with the bandpass you use. If you observe the very red star Betelgeuse, you will detect more light in the *V* bandpass (centered in the green-yellow part of the spectrum) than in the *B* bandpass (centered in the blue part of the spectrum), that is $m_B > m_V$ *(remember, less light means greater magnitude!).* In fact, one can denote the color of a star by listing

$m_B - m_V$ (or more commonly, B–V). [**note:** Subscripts used to denote the bandpass of the observation].

End of Section Questions:

1. How much brighter is the Sun than the full moon?
2. How much brighter are the faintest "naked" eye stars versus the faintest star visible on the Palomar Sky Survey?

Apparent vs. Absolute magnitude

What we have been calling magnitudes up until now are really *apparent magnitudes*. Apparent magnitude describes how bright stars appear to be as seen by your detector. However, they tell us nothing about the intrinsic luminosity (a.k.a. energy output per unit time) of the stars. Why? The apparent brightness of a star depends on two factors: the luminosity of the star and its distance. The Sun is not particularly bright as stars go, but it appears spectacularly bright to us because we live so close to it.

If we want to compare the luminosities of stars using the magnitude system, we have to level the playing field. We therefore define the *absolute magnitude*, of an object as the apparent magnitude one would measure if the object was viewed from a distance of 10 parsecs. We denote absolute magnitude by an upper case *M* and reserve the lower case *m* for apparent magnitude. The absolute magnitude is thus a measure of the luminosity of the object. Consider the stars in table 2 below. Which of these stars is intrinsically the brightest? The supergiant Betelgeuse is the brightest, with an absolute magnitude of -5.6. The sun is intrinsically the dimmest!

Table 2: Apparent and absolute magnitudes of common stars

Object	$\mathbf{m_V}$	$\mathbf{M_V}$
Sun	-26.8	4.83
Sirius	-1.47	1.41
Vega	0.04	0.5
Betelgeuse	0.41	-5.6
Polaris	1.99	-3.2

How can we calculate the absolute magnitude of a star? The easiest way to determine the absolute magnitude of a star is to know its distance. If you know the distance to the star, careful application of the inverse square law of light will allow you to determine the difference between the apparent and absolute magnitude, m – M, also called the *distance modulus*. The next section derives the distance modulus and discusses its application.

End of Section Questions:

3. What are the distance moduli for the five objects in table 2?
4. How much more luminous is Sirius than the Sun? That is, how much more energy does Sirius put per second versus the Sun?

The Distance modulus and Distance

The light from stars (and most other light sources) is not "beamed". Therefore, the energy from the star propagates outward spherically and as such the amount of flux you will receive from a light source of a given luminosity can be written:

$$F = \frac{\text{energy/time}}{\text{area}} = \frac{L}{4\pi d^2} \qquad [3]$$

where *L* is the luminosity of the star and *r* is the distance to that star (such that the light from that star that is reaching you has been spread out over a spherical area $4\pi d^2$). This means we can write a flux ratio for the star at its "true distance" measured in parsecs, d_{pc}, versus 10 pc as:

$$\frac{F}{F_{10pc}} = \frac{L / 4\pi d_{pc}^2}{L / 4\pi (10)^2} = \left(\frac{10}{d_{pc}}\right)^2 \qquad [4]$$

And so, the distance modulus, *m – M,* can be written as (recalling equation [1] in the process):

$$m - M = -2.5 \log_{10} \frac{F}{F_{10pc}} = -2.5 \log_{10} \left(\frac{10}{d_{pc}}\right)^2$$

or, applying our various tricks for dealing with logarithms:

$$m - M = 5 \log_{10} d_{pc} - 5 \qquad [5]$$

Once we know the distance modulus, *m–M,* we can easily calculate the distance to the object, as evidenced by solving equation [5] for distance (in parsecs):

$$d_{pc} = 10^{\frac{m-M+5}{5}} = 10^{0.2(m-M+5)} \qquad [6]$$

This conversion is so easy that many authors give distances to galaxies in units of distance modulus instead of actual space distances, since distance modulus is what they are actually computing.

End of Section Questions:

5. Show that equation [5] is correct, that is, do the necessary "logarithmic tricks" to get equation [5] from the equation shown above it.
6. The Pisces-Perseus Supercluster is at a distance of approximately 76 Mpc (1 Mpc = 10^6 pc). What is the distance modulus of a galaxy in the Pisces-Perseus supercluster? What does this mean?
7. The Andromeda galaxy (M31) has a distance modulus of 24.47. What is its distance?

Surface Brightness

Extragalactic astronomers are often less concerned with the total light coming from an object than the amount of light per unit angular area on the sky. As you might imagine, detectors are limited by the amount of light they can detect per unit area (or pixel, in the case of CCDs). Put another way, if you take a galaxy and spread its light out over 100 times the area on the sky, you have dropped the amount of light hitting the detector per unit area to 1% the original amount, even if the total amount of light emitted by the galaxy is the same.

There are two common units of surface brightness. If you want to refer to properties of the galaxy, you can refer to the total number of solar luminosities per square parsec emitted by the side of the galaxy facing you. This is typically denoted with the upper case *I* for intensity. However we don't directly measure *I* in units of solar luminosities per square parsec, instead we typically measure some flux per unit solid angle on the sky, F/α^2, so it is more common to use units of "magnitudes per square arcsecond" to quantify surface brightness. We can apply the magnitude equation to the flux per unit area to say:

$$-2.5\log_{10}\frac{F}{\alpha^2}$$

where F is the flux of the object and α^2 is the solid angle the object subtends on the sky (in units of square arcseconds). To properly calibrate this so that it is magnitudes/arsecond^2 we have to divide the flux observed by the flux of Vega, such that:

$$\mu = -2.5\log_{10}\frac{F / F_{Vega}}{\alpha^2}$$

Or finally:

$$\mu = m + 5\log_{10}\alpha \qquad [8]$$

where μ is in units of magnitudes per square arcsecond.

End of Section Questions:

8. Show that equation [8] is correct, that is, do the necessary "logarithmic tricks" to get equation [8] from the equation shown above it.
9. Old photographic plates had a limiting surface brightness of about 23 mag/arcsecond^2. Modern CCD detectors can regularly go down to 25 mag/arcsecond^2 with sufficiently long integrations. How much fainter in surface brightness is this?
10. The apparent angular diameter of an object goes as d^{-1} where d is distance. Argue that this means the angular area the object covers on the sky should go as d^{-2}. Based on this argument combined with the inverse square law of light to show that surface brightness is independent of distance!

A final complication: Interstellar reddening

Although we often think of interstellar space as a vacuum, it is in fact filled with tenuous gas and dust. Like a smoke-filled room, the gas and dust along the line of sight to a star dim the starlight by absorbing and scattering the light. We call this effect interstellar extinction. This extinction is stronger at shorter wavelengths, as shorter wavelengths interact more strongly with dust particles. Red light passes through gas and dust more easily than blue light. The reddening of starlight due to the interstellar extinction is known as interstellar reddening. Astronomers often used the terms extinction and reddening interchangeably.

The extinction or reddening to an object is usually given in magnitudes, and denoted by an upper case A. Since extinction is a function of wavelength, a subscript specifies the wavelength for the stated value. A star whose light is dimmed by 1.2 magnitudes when viewed through a V filter would have an extinction of $A_V = 1.2$. This extinction must be taken into account if we are to compute the true absolute brightness or distance modulus of the object.

How do we correct the distance modulus to account extinction? We need to brighten the apparent magnitude to correct for the extinction. Lower magnitudes are brighter, so we want to subtract A_V from the apparent magnitude. The revised equations are thus

$$m - M = 5\log_{10} d_{pc} - 5 + A_V \qquad [9]$$

When dealing with extragalactic sources, we can often "lookup" the extinction due to dust in our Galaxy in the direction of the extragalactic source using the dust maps of Schlegel, Finkbeiner, and Davis (1998, Ap,J **500**, 525) and the extinction curves of Cardelli, Clayton and Mathis (1989, ApJ, **345**, 245) to relate the Schlegel extinctions in B to extinctions at other bandpasses. NED (the NASA Extragalactic Database) provides such a calculator at <*http://nedwww.ipac.caltech.edu/forms/calculator.html*> that can do the computational work for you.

Many excellent links to webpages on photometry and the magnitude system are listed at <*http://outreach.atnf.csiro.au/education/senior/astrophysics/photometry_links.html*>**.**