Astrophysics Problem Set #8

Due Friday, October 30 at 4pm

ASSUMED READING: Before starting this homework, read R&P Chapter 13 as well as the "Magnitudes: Astronomers' Method of Defining Brightness" handout from class.

1. Observational astronomers often use the rule of thumb that to achieve 1% accurate photometry (that is, where the flux is measured to an accuracy of ±1%) requires you to obtain magnitudes accurate to ±0.01. In other words, a 1% change in brightness corresponds to a change of 0.01 magnitude. In equation form, we mean that for small changes in flux such as going from some original flux F_{orig} to some new flux $F_{new} = F_{orig} + \Delta F$, the fractional change is $\Delta F/F_{orig}$ results in a corresponding magnitude change of

   $$\Delta m \approx -\frac{\Delta F}{F_{orig}}$$

   (the negative sign is because in increase in flux results in a decrease in magnitude).

   Verify this approximation is valid for $\frac{\Delta F}{F_{orig}} = 0.01$. **Extra Credit:** Come up with an second-order estimate for the error in the expression above. **HINTS:** The Taylor's series of a natural logarithm of x can be expressed as

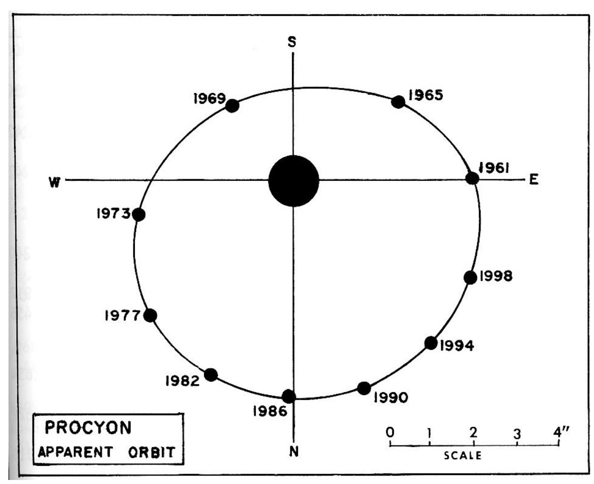
   $$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n} \quad \text{for} \quad |x| \le 1 \quad \text{unless} \quad x = -1$$

   $$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\cdots$$

   and it might be very useful if we can convert from base 10 to natural logarithm using

   $$\ln x = \frac{\log_{10} x}{\ln_{10} e} = \frac{\log_{10} x}{0.4342944819} \longrightarrow \log_{10} x = 0.4342944819 \ln x\,.$$

The textbook gives the example of Sirius as a bright star that is actually a binary star system. Another exam is the star Procyon (aka α Canis Minor) which is the seventh brightest star in the night sky. When examined through a telescope, Procyon is resolved into two stars, a main sequence star called Procyon A (V magnitude 0.34) and a white dwarf companion called Sirius B (V magnitude 10.8). The observed parallax of the Procyon star system by Hipparcos is 0.286 arcseconds.



The observed orbit (centered on Procyon A) is shown to the right.[1] Note that this plot is centered on Procyon A and does **NOT** show the motion of the stars around their mutual center of mass.

After correcting for projection effects, the semi-major axis of Procyon B's orbit relative to Procyon A is 4.27 arcseconds on the sky.[2] The orbital period is 40.8 years. Procyon A's orbit appears to have a semi-major axis of 1.20 arcseconds relative to the center of mass. Please answer the following questions about the Procyon star system.

2. Assuming they emitted enough X-rays to be visible at that wavelength, what would you see if you pointed the *Uhuru* X-ray explorer satellite with a 0.5° angular resolution at Procyon A and B. Assume for the sake of argument that both Procyon A and B emit equal amounts of X-rays. Compare this with what you would see with the *Chandra* X-ray satellite currently in operation, which has a 0.5 arcsecond angular resolution? *Explain your reasoning.*

3. How many times brighter is Procyon A than Procyon B?

4. What is the combined magnitude of Procyon A and B? That is, when you combine the light from the two stars (such as what you see with your naked eye), what is the magnitude of their combined flux? How different is it from the magnitude of Procyon A alone? Did you expect this?

5. Could Procyon A and B be resolved with a small 10-cm (4 inch) diameter telescope at visible wavelengths (λ~500 nm) at a good site (assume seeing ~ 0.25 arcseconds)? Does this explain why Sirius B was not observed until 1896 by John Martin Schaeberle using a much larger telescope? If not, what might explain the late discovery of this visual binary?

6. What is the distance to the Procyon star system (in parsecs) and what is the semi-major axis of their orbit in AU?

7. Estimate the absolute magnitudes of Procyon A and B. Use this to compare their luminosity to that of the Sun which has an absolute V magnitude of 4.80.

8. The color index B-V of the two stars are 0.40 (Procyon A) and –0.22 (Procyon B). Assuming both are blackbody spectra, which object is hotter? *Explain your reasoning.*

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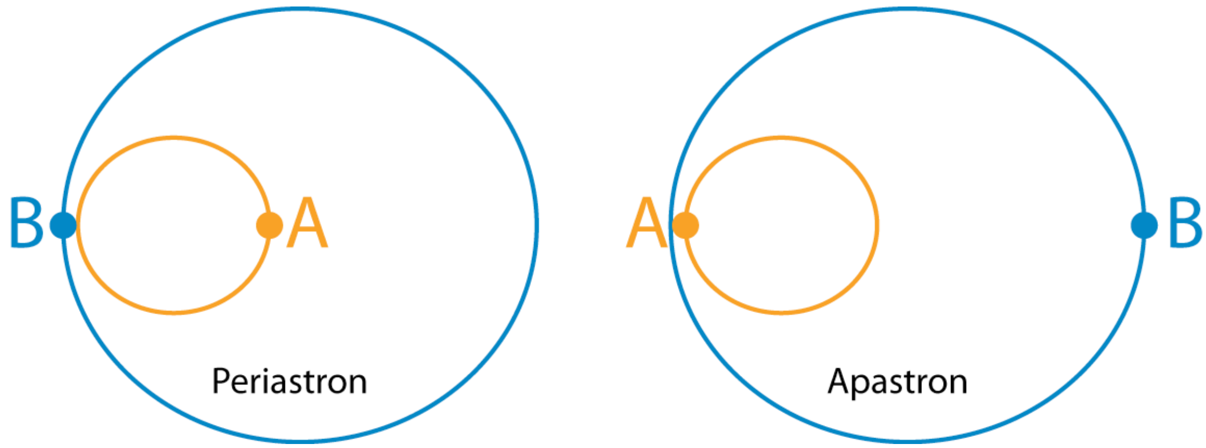
[1] This figure is taken from *Burnham's Celestial Handbook (Volume One)* by Robert J. Burnham , Jr. 1978, Published by Dover. This orbital sketch will not perfectly match the refined orbital parameters presented here, but it is a reasonable visualization.

[2] This orbital parameters cited here are from "A Redetermination of the Mass of Procyon" by Girard, R.M., *et al.* (2000) *The Astronomical Journal*, **119**, 2428. You can look at *http://spiff.rit.edu/classes/phys440/lectures/fix_tilt/fix_tilt.html* for the entire, tedious, procedure on how the true orbit is be determined from the apparent orbit on the sky.

9. Assuming Procyon A and B are effectively at the same distance from us, what can you say about the relative radii of Procyon A and B? **NOTE:** I am not looking for an explicit calculation, I am looking for a statement about the relative sizes of these stars.



10. The figure above shows the orbits of Procyon A and B in the center of mass frame, deprojected so that we are look straight down on the system. From this figure you can see that the orbit of Procyon A relative to the center of mass is smaller than that of Procyon B,. This is because Procyon A is more massive than Procyon B. If the proper deprojected semi-major axis of Procyon A relative to the center of mass is 1.23 arcseconds, what are the masses of Procyon A and B? **HINT:** The semi-major axis of Procyon B must be 3.04 arcseconds, since the sum, 1.23 + 3.04 = 4.27 arcseconds, is be the semi-major axis of Procyon B relative to Procyon A as quoted above. Use this as the semi-major axis of the system.