Astrophysics Problem Set #6

Due Friday, October 14 at 4pm

In the homework problems below, "R&P" refers to your textbook. You may work together on this problem set, but all work presented here must be your own. **You must clearly acknowledge any people you collaborated with.**

ASSUMED READING: Before starting this homework, read R&P Chapter 5. You may also need to review your intro Physics textbook section on light waves.

1. There are two commonly used terms for quantifying a light source. There is the *flux* which has metric units of Watt m^{-1} or Joules sec^{-1} m^{-1}. We can also describe a light source using *luminosity* which has metric units of Watts. Explain in English the difference between the flux of a star and its luminosity.

2. Only about 10% of the light hitting the top of Earth's atmosphere with a wavelength of 3 μm makes it to the surface. What is the optical depth of the Earth's atmosphere at 3μm?

3. Deduce an approximate expression for Planck's radiation law, $I_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1}$ (equation 5.86).

   a. at high frequencies ($h\nu \gg kT$). This is the called the Wien distribution $I_\nu \approx \frac{2h\nu^3}{c^2}e^{-h\nu/kT}$ where $h\nu \gg kT$. **HINT:** Start with simplifying the exponential term for cases where $h\nu/kT \gg 1$.

   b. at low frequencies ($h\nu \ll kT$). This is called the Rayleigh-Jeans distribution

   $$I_\nu \approx \frac{2kT}{c^2}\nu^2 \quad \text{where } h\nu \ll kT.$$

   You may find the Taylor's series approximation that $e^{h\upsilon/kT} \approx 1 + \frac{h\upsilon}{kT}$ when $h\nu/kT \ll 1$ useful.

   c. Find the wavelength as a function of temperature when neither approximation above is appropriate, that is when $h\upsilon/kT = 1$. Do you notice any similarity between the expression for this wavelength and Wien's Law, $\lambda_p = \frac{2.898\times10^{-3}\, m\cdot K}{T}$?

4. [R&P 5.6 on a diet]

a. For the Planck function expressed as a function of wavelength

$$I_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda kT}\right)-1}$$ (see equation 5.90),

derive the expression for Wein's law, the most probable wavelength λ_p at a given temperature T? **NOTE:** At some point you will end up with a function of the form $5 = x\left(1-e^{-x}\right)^{-1}$ where you can solve for "x" numerically either via a computer program like *Maple* (using the 'fsolve' command) or by trial and error. Please be clear how you solve for "x".

b. For what range of temperatures does λ_p fall in the visible range of the electromagnetic spectrum? **NOTE:** Visible light runs from about 350 to 750 nm.

5. Consider two stars <u>of the same size</u> which emit light as blackbodies.

a. How much more energy is emitted by a star at 12000K than one at 3000K? *Clearly explain your reasoning.* **[HNT:** With the information given, you can NOT work out the total energy output for each star. You don't need to solve this problem!]

b. What is the predominant color of each star in part (a)? Use Wien's Law to express your answer in wavelengths as well as words.

c. Why can you see the element of an electric stove become orange if it's temperature is considerably less than 1000K?

6. [R&P 5.8] If an incandescent light bulb has a luminosity L = 60 W and a filament temperature of T = 2900 K, what must be the surface area of its filament? If the filament consists of a cylindrical wire with diameter $d = 4.6\times10^{-5}$ m (as in a standard General Electric 60 Watt, 120 volt bulb), what is the length of the wire? [HINT: Assume the wire is indeed cylindrical with a surface area equal to $A = \pi dl$ where l is the length of the wire.]

7. One of the major difficulties with gamma-ray astronomy is the high angular resolution limit (poor resolving power), with angles less than 2˚ being unresolved even for the best instruments. Nevertheless, gamma-ray images of our galaxy have been produced with satellite data (See http://apod.nasa.gov/apod/ap971105.html for an example image). Discuss the difficulty in interpreting these images in light of the high angular resolution limits (poor resolving power).