Astrophysics Problem Set #5 (Journal Reading) Solutions [30 points possible]

1. **Journal Reading:** Please download "Measurement of the Diameter of Alpha Orionis with the Interferometer" (Michelson, A.A. and Pease, F.G. 1921, Astrophysical Journal, 53, 249) from the electronic handouts page on our course website. In this article, Michelson measures the diameter of Betelguese using an interferometer of his own design, something that hadn't been done before and wasn't pulled off by anyone else for over 20 years. This is a relatively short paper (11 pages), but again, my goal is not for you to memorize the paper, but for you to pay attention to how Michelson and Pease pulled off the measurement of the diameter of a star. After reading the article answer the following questions:
   a. What is interferometry? Why is it difficult to set up an interferometer? To answer these questions, you may have to look this <u>outside</u> of the article. Be sure to cite your sources. **NOTE:** Wikipedia or any other website is NOT an acceptable source, but using it to find "refereed sources," such as textbooks or peer-reviewed articles, is acceptable.

      [10 points possible] *Interferometry is a technique in which light coming from the same light source along two (or more) different paths is brought together to form an interference pattern. Since the light beams will have travelled slightly different distances, when they are brought together they could be "in phase," if the path length difference of the two paths is equal to a unit number of wavelengths. Otherwise, the light waves will be out of phase and will interfere with one another.*

      Consider a laser beam hitting a pair of closely spaced slits will produce an interference pattern on a screen. Imagine running the entire process in reverse. Light comes off the screen toward the two slits. If light is from a location on the screen where there was constructive interference, then when the light passes through the two slits, you will get a coherent 'beam' . If the light comes from a place on the screen where there was interference, the light we destructively interfere when going back through the two slits, and produce no beam of light. In this analogy, the screen is the sky and the slits are our telescopes. Since the double slit-interference pattern is narrower than the single-slit diffraction pattern, properly designing an interferometer out of two telescopes will produce a much narrower 'beam' on the sky, meaning we can see tinier details than we could through a single telescope, although we have to sacrifice a lot of light to do it (meaning we can only see brighter objects).

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b. The smallest detail a telescope can make out in ideal conditions is governed by the "Airy disk" equation

$$\sin\theta = 1.22\frac{\lambda}{D}.$$

where λ is the wavelength of the observation, D is the diameter of the telescope's primary mirror/lens, and θ is the angular size of the smallest detail that can be seen. We assume wavelength and diameter are in the same units. For small angles **expressed in radians**, this can be written:

$$\theta \approx \sin\theta \longrightarrow \boxed{\theta \approx 1.22\frac{\lambda}{D}}.$$

Use this expression to determine the diameter of a telescope with enough resolution to measure the diameter of Betelguese using an optical telescope centered in the red (λ = 700 nm = 7000Å). Based on your answer, does it make sense to use an interferometer instead of a large telescope[1]? **NOTE:** Remember that to measure the diameter of Betelguese, you need to have an angular resolution sufficiently smaller than Betelguese to make out its diameter.

[10 points possible] *Michelson and Pease (1921) cite an angular diameter for Betelguese's stellar disk of 0.047 arcseconds which corresponds to*

$$0.047\,\text{arcsec}\frac{1\text{ (radian)}}{206265\,\text{arcsec}} = 2.279\times10^{-7}\text{ (radian)}.$$

[I forgot to mention the necessity to convert to radians in the small angle approximation, so I will be generous if the student made that incorrect assumption. I added the "boldfaced" mention of radians to the problem stated above to say "what I meant."].

Ignoring the complication of atmospheric seeing affecting the observed angular resolution,[2] the angular resolution of a telescope is given by the Airy disk equation, which we can rearrange to solve for the necessary diameter D of the telescope

$$D \approx 1.22\frac{\lambda}{\theta} = 1.22\frac{700\times10^{-9}\,m}{2.279\times10^{-7}} = 3.7m$$

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[1] The largest optical telescopes at the time of Michelson and Pease's work were about 2.5 meters in diameter. The largest today have 10 meter mirrors, although some exploit interferometric techniques with multiple mirrors to achieve greater resolution.

[2] The best optical seeing conditions on the ground typically produce a seeing disk (the minimum resolution that can be resolved with a single aperture) on the order of 0.25 arcseconds at the very best sites in the world under the best observing conditions!

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To see twice as much detail, I would need a telescope twice as big. Well within reach of the telescopes we are constructing today? Surprised?

Based on this answer, you might think today we would just used a big telescope, but in fact, the problem is that we live under an atmosphere that 'blurs' the light coming down from space. This was, in fact, a large part of the reason why Michelson and Pease's 1921 achievement wasn't reproduced for another 45 years, and then only by switching to radio waves, where the atmosphere is almost completely transparent and not a factor. Modern telescopes are deploying adaptive optics systems which sample the seeing disk and make adjustment to the shapes/orientation of the mirrors in the telescope at rates on the order of 100 Hz in order to compensate for the atmosphere. These systems are now reaching resolutions on the order of the optical limit described by the Airy disk, even for large telescopes.

c. Write up of a question Michelson and Pease answered in their article and your best attempt to understand their answers based on the paper.

   [10 points possible] *This problem will be graded based on the depth of question you selected and the thoroughness of the solution you present.*

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