Astrophysics Problem Set #4

Due Friday, Sept. 25 at 4pm

In the homework problems below, "R&P" refers to your textbook. You may work together on this problem set, but all work presented here must be your own. **You must clearly acknowledge any people you collaborated with.**

ASSUMED READING: Before starting this homework, read R&P Chapters 4 (start to finish) and Chapter 5.1 to 5.3. You may also need to review your intro Physics textbook section on light waves.

1. **Tidal Forces at a Black Hole.** Last week we talked about a process called *spaghettification* that occurs in which material falling toward a black hole is tidally shredded. I want you to consider the difference in tidal shredding for material falling into a small black hole versus the supermassive black holes at the center of galaxies.
    - **a.** Let's assume you are falling into a 3 solar mass black hole (estimated Schwarzschild radius[1] of about 9 km). Assuming you are 1.5 meters tall and 70 kg in mass and falling in feet first, at what distance would the gravitational force on your feet exceed the gravitational force on your head by 10 kN. You can assume at this point you would be fatally *spaghettified.* Compare this distance to the black hole's Schwarzschild radius?
    - **b.** Now consider instead a trip toward the supermassive black hole at the center of our galaxy, which has an estimate mass of about 2.5 million solar masses (estimated Schwarzschild radius of about 7.5 million km). This is roughly a million-fold increase in the mass of the black hole compared to (a). How much does this increase the distance at which you will be *spaghettified.* Would you die by *spaghettification* before falling within a supermassive black hole's Schwarzschild radius? **HINT:** You can do this problem without re-doing the entire calculation you did in part (a) as long as you consider how this *spaghettification* varies with mass of the black hole.

[1] The "Schwarzschild radius" is the distance from the center of the Black Hole at which the escape velocity equals the speed of light. Once inside this radius, there is no way to exit the black hole that we are aware of.

2. **Basics about Light:** This problem covers material I assumed you know for this class regarding the wave nature of light. If you haven't already seen this, talk to me. The material can easily be covered in a few minutes in my office.

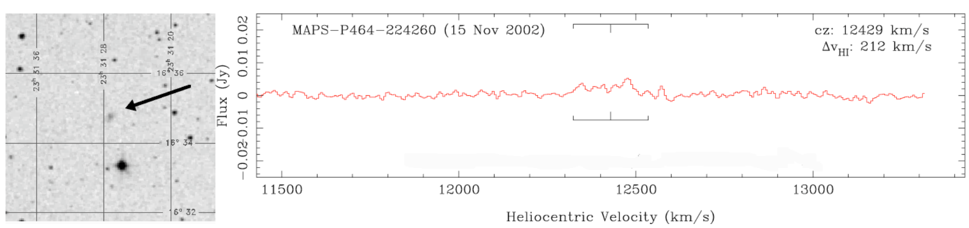
   a. The hydrogen hyperfine transition spectral line is extremely important as this spectral line is emitted by neutral hydrogen (called "HI", that is "H-one," by astronomers). This allows us to trace hydrogen gas in both Galactic and Extragalactic contexts with a simple radio telescope. The hydrogen hyperfine transition spectral line has a rest frequency of 1420.405752 MHz. What is the wavelength of this line (with proper significant figures)? **HINT:** The speed of light is (exactly) 299792458 m/s (assume no error in this number).

   b. The Doppler effect occurs when a light source is moving relative to the observed such that light emitted with wavelength λ_0 is seen with wavelength λ by an observer. The classical expression for Doppler shift in terms of wavelength is written:

   $$\frac{\lambda}{\lambda_0} \approx \left[1 + \frac{v}{c}\right]$$

   where v is the relative velocity away from the observed (and is negative for approaching object) and c is the speed of light. Show that in terms of the emitted frequency of light, υ_0, and observed frequency of light, υ, this expression can be written

   $$\frac{\upsilon}{\upsilon_0} = \frac{1}{1 + \frac{v}{c}}.$$



   c. Using the Arecibo radio telescope, your professor barely detected HI in a low surface brightness galaxy, MAPS-P464-0224260, receding from us (due to the expansion of the Universe) at a relative velocity of 12429 km/s. An image of the galaxy and the spectrum (in velocity coordinates) is shown. What was the observed central frequency of the HI spectral line [this is the same line whose rest frequency is given in part (a)]?

3. The estimated speed of rotation of MAPS-P464-0224260 (the galaxy in the last problem) was 212 km/s about the center of the galaxy [as indicated in the plot of the spectrum above]. Estimate the frequency resolution we need to clearly see the the Doppler broadening of the spectral line, $\Delta\upsilon/\upsilon_0$. **NOTE:** In astronomy, $\Delta\upsilon$ represents (roughly) the width from the line edge to the line center. You do <u>not</u> need to know the observed central frequency of this line (which you figured out in part (c) of the last problem. Just explain why

$$\frac{\Delta\upsilon}{\upsilon_0} = \frac{\upsilon_{edge} - \upsilon_{center}}{\upsilon_0} \approx -\frac{v_{rotation}}{c}$$

for velocities much less than the speed of light (you will have to explain the reasoning well) and then apply the expression.

4. **Playing with the Doppler Effect Equations:** The fully, relativistically correct expression for the Doppler Effect is:

$$\frac{\lambda}{\lambda_0} = \left[\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right]^{1/2} = \left[1+\frac{v}{c}\right]^{1/2}\left[1-\frac{v}{c}\right]^{-1/2}$$

but it can be rather awkward to work with (especially if you want to solve for the velocity given the observed wavelength. If the velocities are not relativistic (that is $\frac{v}{c} \ll 1$), you can use "classical" expression for the Doppler shift

$$\frac{\lambda}{\lambda_0} \approx \left[1+\frac{v}{c}\right]$$

Show that the relativistically-correct expression for the Doppler shift reduces to the classical expression when $\frac{v}{c} \ll 1$. **HINT:** Use the simple form of the binomial theorem which states that:

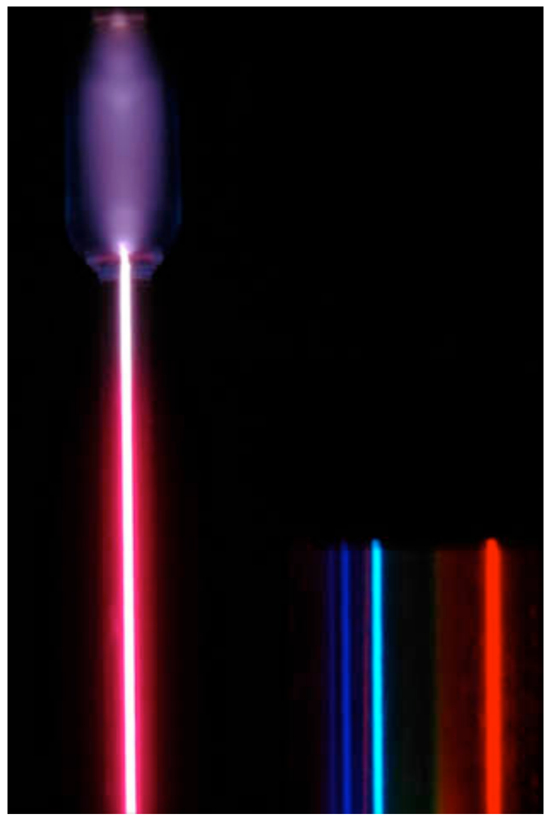
$$(1 \pm x)^n = 1 \pm nx + n(n-1)\frac{x^2}{2!} \pm n(n-1)(n-2)\frac{x^3}{3!} + \cdots \qquad \text{when } x^2 < 1.$$

5. Given the expression for the Bohr Model of the wavenumber of electron transitions between energy levels in the hydrogen atom,

$$\lambda = \frac{hc}{E_{photon}} = \frac{hc}{\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{m_e}{2\hbar^2}} \left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]^{-1} = 911.6\text{Å}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]^{-1} .$$

   a. Compute the spectral lines of hydrogen in the visible part of the spectrum and then accurately sketch the emission line spectrum of hydrogen, labeling the lines based on their transition. **HINTS:** The visible portion of the spectrum runs from about 400 nm to 700 nm. All the visible light electron transitions in Hydrogen are Balmer lines (transitions into the n=2 energy level in hydrogen).

   b. Compare the spectrum you sketched to the following image of an actual hydrogen spectrum (from the *HyperPhysics* website) created by using a 5000 volt potential to excite electrons in a hydrogen spectral tube (on the left hand side of the picture) and shining the light through a 600 line/mm diffraction grating in order to separate the spectral lines (shown to the right). Which lines in the image correspond to which lines in your computations in part (a). Explain your reasoning.



6. [**Extra Credit]** The emission line of He II at 468.6 nm corresponds to what electron transition? **HINTS:** This seemingly simple question has an annoying wrinkle. You will end up with a single equation with two unknowns. This is not mathematically solvable, but the number of possible values of the two unknowns is relatively restricted. Playing around with *Excel* might make the task of finding the right transition a bit less tedious, although insight will be more powerful than *Excel* any day. [**Note:** This problem was originally from the Zeilick & Gregory, another astrophysics textbook we are not using.]