

Astrophysics Problem Set #3

Due Friday, Sept. 18 at 4pm

In the homework problems below, “R&P” refers to your textbook. You may work together on this problem set, but all work presented here must be your own. **You must clearly acknowledge any people you collaborated with.**

ASSUMED READING: Before starting this homework, read R&P Chapters 3 and 4 (start to finish).

1. Explain (in words) how the Virial Theorem for gravitational systems

$$2\langle K \rangle + \langle U \rangle = 0$$

allows you to use the redshift-measured velocities for galaxies in a galaxy cluster to determine the mass of the galaxy cluster. I am not asking you to actually do the calculation, but simply to explain essentially how you obtain $\langle K \rangle$ and $\langle U \rangle$ and how they are related to velocities and the masses.

2. Every couple of years, an astrologer tries to argue for the scientific legitimacy of astrology (versus astronomy) by claiming it is the tides of the planets on the human body that affect a person’s life (why the effects would depend on the tidal forces the moment you were born is unclear to me).
 - a. Explain why we would expect the tidal forces on a human being less than 20 inches long at birth to be rather unimportant compared to tidal forces on the Earth. I want you to explain this in words, without resorting to explicit calculations. You may refer to formula if you have to, but concentrate on the relationships between variables, **DO NOT CALCULATE ANYTHING.**
 - b. Consider my twin 6-1/2 year olds, Zakari and Carolina. They were born with a weight of about 1.7 kilos each and a length of about 43 cm each. Compute the maximum tidal forces on one of my twins due to:
 - i. The moon. Assume it was at its average distance from Earth.
 - ii. The planet Jupiter. To maximize the tidal force, assume Jupiter was as close to the Earth as it can get (I leave it to you to explain how you determine what this closest approach distance is).
 - iii. The obstetrician who delivered them, an 60 kilo female obstetrician. Assume an average distance of about 1 meter between her and my twins during the birth.
 - c. What do the results of part (b) mean for the idea that planetary tides could significantly affect a birth, at least as far as scientifically “justifying” astrology is concerned?

3. Let's consider the idea of "spaghettification" near a black hole. For our purposes, a "black hole" is just an extremely dense concentration of mass. As long as the local escape velocity near the black hole is not a significant fraction of the speed of light, we can use Newton's Law of Gravity to describe the gravitational effects near a black hole. Imagine you are falling into the black hole known as Cygnus X-1 orbiting the blue supergiant HDE 226868. Imagine the huge amount of X-ray radiation in this environment hasn't already killed you. This black hole has an estimated mass of 8.7 solar masses and an estimated diameter of 26 km (as I said, very dense). Estimate how close you could get to the black hole, falling feet first, before being fatally "spaghettified" by the tidal forces. Since I am not an expert on human physiology, I will leave it up to you to clearly state how you estimated what would be a 'fatal' tidal force.

4. The X-rays in Cygnus X-1 (see last problem) are produced by the material falling into the black hole. Specifically, due to conservation of angular momentum, material doesn't fall directly into the black hole, instead it forms an accretion disk of material. Assume the material has been disrupted so that it is essentially a bunch of tiny particles orbiting the black hole in a 'gaseous' state.
 - a. Assuming circular orbits, explain in words what happens to the orbital velocity of material in the accretion disk as it approaches the black hole.
 - b. Using your answer from part (a), explain (in words) why material in the accretion disk drops into lower orbits over time. **HINT:** For it to drop into a lower orbit, it must lose energy. Think of some way that these particles can interact where there is a net loss in energy. **HINT #2:** Rub your hands together quickly, what is produced and why?
 - c. Given your answer to part (b), explain why computing how fast the orbital speed varies as a function of radius will tell you where the accretion disk is hottest. Compute the necessary derivative to explain where the accretion disk will be the hottest. For reasons we will discuss later, this is where the X-rays in Cygnus X-1 are produced.

5. **[R&P 4.2 Tweaked for context]** Most of the exoplanets discovered so far have been Jupiter-sized or larger and in orbits smaller than 1AU! How close to the Sun could the planet Jupiter come without suffering tidal disruption? **NOTE:** We are ignoring the possibility of the heat from the Sun heating the planet enough to cause its gases to escape, so a Jupiter-sized planet outside the Roche limit might still be in for a destructive experience (I'll admit, I am not sure if the heat from the Sun is enough, maybe a future problem will explore this).