Astrophysics Problem Set #2

Due Friday, Sept. 11 at 4pm

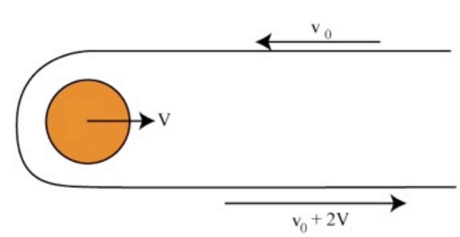
In the homework problems below, "R&P" refers to your textbook. You may work together on this problem set, but all work presented here must be your own. **You must clearly acknowledge any people you collaborated with.**

ASSUMED READING: Before starting this homework, read R&P Chapter 3 (start to finish).

1. **[R&P 3.2 tweaked]** The asteroid Eros is seen in opposition from the Earth once every 847 days.
    a. What is the sidereal period of Eros?
    b. What is the length *a* of the semi-major axis of Eros' orbit?
    c. The eccentricity of the orbit of Eros is $e = 0.223$. Clearly show if Eros ever comes within 1AU of the Sun? Why is this an interesting question to consider?

2. There have been several recent missions to send spacecraft to the outer solar system. One of these missions is the *New Horizons* mission to Pluto, which launched in January 2006 and expected to pass by Pluto in July 2015.
    a. Estimate is the semimajor axis of the least-energy elliptical orbit of a space probe from Earth to Pluto (Pluto is near perihelion at a distance of about 31 AU) and how long would such a mission take?
    b. How does this compare to the *New Horizons* mission flight time and what does this tell you?

3. In order to achieve the speed necessary to get to Pluto so quickly, the *New Horizons* space probe engaged in a gravitational assist maneuver around Jupiter. Gravitational assist works because we use the gravity of the **moving** planet to gain energy relative to the Sun, thus speeding up the spacecraft relative to the Sun. Let's dissect this a bit:
    a. Imagine for the sake of argument that a spacecraft approached a planet "head on" as shown in the drawing to the right. Representing planetary parameters with capital letters and spacecraft parameters with lower case, show that taking into account conservation of kinetic energy (assuming same inbound and outbound "height" from the planet)::



"head on" collision in "Solar System" Reference Frame

$$\frac{1}{2}MV_0^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}MV_1^2 + \frac{1}{2}mv_1^2$$

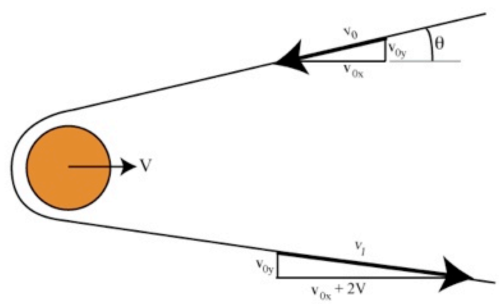
and conservation of momentum:

$$MV_0 - mv_0 = MV_1 + mv_1$$

show that the velocity of the spacecraft after passing the planet must be $v_1 \approx 2V + v_0$ assuming $m \ll M$. **HINT:** This can be surprisingly nasty to do even though it is just algebra. This is really a "purely elastic collision" problem, in case you want to research it. **HINT 2:** Try re-writing the conservation of energy and momentum conditions moving each objects velocities to one side and exploiting the algebraic equality $(a^2 - b^2) = (a+b)(a-b)$ to simplify things. Solve for V_1 and then use this to eliminate it and solve for v_1.

b. Considering part (a), how much does the speed (<u>not</u> velocity) of the space probe change in the reference frame of the planet? Surprised?

c. **[Extra Credit]** Let's do this same problem in two-dimensions (as shown in the figure to the right). In this case, only the component of the velocity parallel to the planet's motion is changed. Show that in a full 2-D case, the outbound <u>speed</u> is given by



2-D collision in "Solar System" Reference Frame

$$v_1 = (v_0 + 2V)\sqrt{1 - \frac{4v_0 V[1-\cos\theta]}{(v_0+2V)^2}}$$

where θ is the angle of approach. [**HINT:** Despite being "extra credit," this problem is just a vector components problem. Apply the Pythagorean theorem to determine an expression for the total outbound velocity v_1 and work from there toward the expression above.]

d. Now let's use the expression derived in part (c) in a real gravitational assist problem. Assuming the initial speed of the *New Horizons* space probe after its major fuel burns was 23 km/s and it approached Jupiter 'from the side' at an angle of 103° and the orbital velocity of Jupiter is 12 km/s. What should its speed be when on the outbound leg of its flight after the gravitational assist?

4. **Journal Reading:** Please download "On the Masses of Nebulae and of Clusters of Nebulae" (Zwicky, F. 1937, Astrophysical Journal, 86, 217) from the electronics handouts page on our course website. The article discusses various methods used by Fritz Zwicky to determine the masses of individual galaxies and clusters of galaxies (referred to at the time as "nebulae") . This paper saw the application of the virial theorem to the problem. It is a longish paper (29 pages), so I don't expect you to learn all the material in it in one pass, but pay attemtion especially to the sections on the application of the viral theorem to the problem and the results. Write up of 2 questions and your best attempt at answers based on the paper. Turn them in with this problem set.