

# **Astrophysics 410 Mid-Term #1 Study Guide**

## **Date of Exam: Tuesday, October 6**

The first Astrophysics 410 Mid-Term exam will occur in class on Tuesday, October 6. You will be allowed the entire 75 minutes for the exam, although I will be aiming for it to be 60 minutes in length.

### **Rules:**

1. Closed Book and Closed Note, but you will be allowed one side of an 8 ½ by 11 inch sheet of paper for notes. I will provide equations on the exam as well, but often, the process of making the notes can be helpful for learning.
2. You will be allowed a calculator and expected to use it.
3. If you are stuck because you can't remember a formula, I will assure you that you can ask me... I am not big on formula memorization.

**Format of Exam:** There will be between three and five problems on the exam. They may contain multiple parts.

- There will be conceptual problems where I expect you to answer in short answer form with little algebraic manipulation. These will involve the physical concepts behind several of the topics we have covered. *e.g.* - Is it appropriate to invoke the Virial Theorem to determine the mass of a young (few million year old) star cluster still forming from its gas cloud. Why or why not?
- There will be some problems that will involve some algebra. These will probably use similar concepts to what was on the problem set problems, although I will avoid the more involved mathematics on the exam.
- A sample exam is attached so you can review it to get an idea about the style and difficulty of the midterm exam 'algebraic' questions. But keep in mind
  - the format that year didn't include conceptual problems. **This year's exam will include conceptual problems!**
  - **Do NOT assume the topics covered on the old exam are the same as this year's exam. There is some material we covered this year that is not shown on the old exam that is fair game for this year.**

## Topics on this Exam (with some notes):

You can expect the problems on the mid-term exam to cover the concepts on the first five problem sets (outlined below).

### A. Gravity and its Applications

1. **Celestial Mechanics:** There will likely be at least one question related to basic Celestial Mechanics as covered in the first few problem sets and the lectures related to them.
  - a. Know what the different kinds of orbital periods are and to what objects they apply. Specifically **Synodic Period** and **Sidereal Period**. Know how Sidereal periods of planets can be determined based on knowledge of the Synodic period. I don't mean memorizing the equations, I mean knowing how it is done.
  - b. Know what conjunction, quadrature, and other descriptions of observed positions of the planets mean, both as seen from Earth and in terms of the positions of the worlds in their orbits.
  - c. The properties of Elliptical Orbits. Be able to determine orbital properties based on some known orbital properties from ellipticity, semi-major axis, orbital period, etc.
  - d. **Kepler's Laws:** Notably understand how Newton's version of Kepler's Third Law related the period, semi-major axis, and masses of two objects in orbit around one another are related.
  - e. **Newton's Laws of Motion and Gravitation** are assumed knowledge at this point. They are useful for verifying Kepler's Laws are physically supported.
  - f. What is the **virial theorem** and in what situations does it apply?
2. **Differential Gravitational Forces**
  - a. Understand where tides come from and be able to explain them clearly.
  - b. Be able to compute tidal forces given the appropriate masses, object radii, and object separations.
  - c. Know what is meant by the Roche limit. Again, not just the equation but the actual physical principles invoked to derive it.

### B. Light and its interactions with Matter

1. **Wave Equation:** Be able to use the wave equation to compute wavelength given frequency and visa versa for electromagnetic radiation.
2. **You should have a rough idea of the typical wavelengths of the various types of electro magnetic radiation.**
3. **Doppler shift:**
  - a. Know what Doppler Shift and how one could compute the shift in radiation due to the motion of a light source relative to us.

- b. Know which situations require you to use the relativistic Doppler shift equations and when you can use the non-relativistic approximations.
- c. Given the shift in a spectrum, be able to compute the speed of the light source.

**4. Flux vs. Intensity**

- a. Know what flux is versus intensity.
- b. Be able to relate the observed flux and distance to an object to its luminosity.

**5. Spectral Lines (interaction of Light and Matter)**

- a. Know the Bohr Model and its relationship to the generation of spectral lines, both specifically in hydrogen and in other atoms.
- b. Know Kirchoff's Laws and be able to explain why a particular gas is seen to have emission lines or absorption lines.
- c. Know how to use the equation of radiative transfer to determine the amount of attenuation in intensity of light passing through a material. Know what number density, column density, and optical depth are and how they are related.
- d. Be able to compute the relative populations of two energy levels in an atom at a given temperature using the Boltzmann equation.
- e. Be able to compute the ionization fraction for a given atomic population at a given temperature using the Saha equation.
- f. How are the Boltzmann and Saha equations combined to explain the spectra of atoms (we discussed the Balmer lines of Hydrogen in this context).

# Astrophysics 410 Midterm Exam (edited for review in Fall 2009) Fall Semester 2007

There are four problems on this exam (one problem is not shown because it covered material we are not covering in the Fall 2009 exam). You must properly complete two of the first three problems in addition to problem four for full credit. I will select your best two answers from problems 1-3 in case you do all three. Please box your final answers.

## Potentially Useful Constants

Gravitational Constant:	$G = 6.674 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$
Planck's Constant:	$k = 1.380 \times 10^{-23} \frac{J}{K} = 8.617 \times 10^{-25} \frac{eV}{K}$
Boltzmann's Constant:	$h = 6.626 \times 10^{-34} J \cdot s = 4.136 \times 10^{-15} eV \cdot s$
Stefan-Boltzmann Constant:	$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$
Speed of Light:	$c = 299792458 \frac{m}{s}$
Mass of the Sun:	$M_{Sun} = 2 \times 10^{30} kg$
Luminosity of the Sun:	$L_{Sun} = 4 \times 10^{26} W$

## Potentially Useful Units

1 electron volt:	$1eV = 1.602 \times 10^{-19} J$
1 Astronomical Unit:	$1AU = 1.496 \times 10^{11} m$
1 Parsec:	$1pc = 206265AU = 3.086 \times 10^{16} m = 3.262 \text{ light-years}$
1 year:	$1 \text{ yr} = 3.156 \times 10^7 s$

Potentially Useful Relationships (Not a complete list)

Kepler's Third Law:	$P^2 \propto a^3$
Newton's Version of Kepler's Third Law:	$P^2 = \frac{4\pi^2}{G(m_1+m_2)} a^3$
Vis Viva Equation:	$v^2 = G(m_1 + m_2) \left[ \frac{2}{r} - \frac{1}{a} \right]$
Circular Orbit Velocity:	$v_c = \sqrt{\frac{GM}{R}}$
Escape Velocity:	$v_e = \sqrt{\frac{2GM}{R}}$
Tidal Forces:	$d = 2.44 \left( \frac{\rho_M}{\rho_m} \right)^{1/3} R$
Roche's Limit:	$\frac{\Delta F}{\Delta R} = -\frac{2GMm}{R^3}$
RMS Speed of Gas Particles:	$v_{rms} = \sqrt{\frac{3kT}{m}}$
Wave Equation:	$\lambda\nu = c$
Doppler Shift (Non-Relativistic):	$\frac{\Delta\lambda}{\lambda_0} = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c}$
Doppler Shift (Relativistic):	$\frac{\lambda}{\lambda_0} = \frac{1+v/c}{\sqrt{1-v/c}}$
Energy per Photon:	$E = h\nu$
Inverse Square Law for Light:	$F = \frac{L}{4\pi R^2}$
Hydrogen Energy Levels:	$E_n = -R' \frac{1}{n^2}$ where $R' = 13.6eV$
Boltzmann's equation:	$\frac{N_B}{N_A} = \frac{g_B}{g_A} \exp [-(E_B - E_A)/kT]$
Saha's equation:	$\frac{N_{n+1}}{N_n} = \frac{A(kT)^{3/2}/N_e}{g_A} \exp [-\chi_n/kT]$

1. [40 points]  $\epsilon$  Eridani ( $\epsilon$  Eri) is the third closest star outside the solar system visible without a telescope. It happens to be one of the relatively few stars known to have multiple planets orbiting it. The two (known) planets orbiting  $\epsilon$  Eri are  $\epsilon$  Eri b with an orbital period of  $\sim 6.85$  years and a semi-major axis of 3.39 AU and  $\epsilon$  Eri c with an orbital period of  $\sim 280$  years and a semi-major axis of  $\sim 40$  AU. [**HINT:** Part (c) can be done without doing parts (a) and (b).]
  - (a) What would be the orbital period of an object in our solar system with a semi-major axis of 3.39 AU. You should assume the object is planetary in mass, that is MUCH less than the mass of the Sun.
  - (b) Comparing the orbital period you just derived to the orbital period of  $\epsilon$  Eri b, what can you say about the mass of the star  $\epsilon$  Eri?
  - (c) Estimate the mass of  $\epsilon$  Eri (in units of solar mass) based on the orbital information given for  $\epsilon$  Eri b.

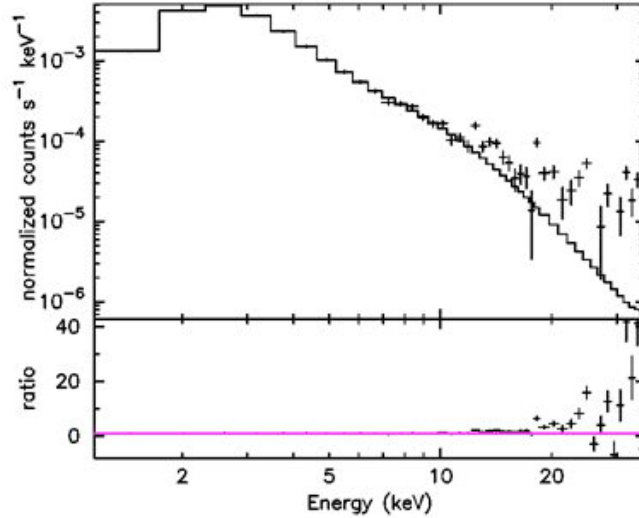


Figure 1: X-Ray Spectrum of Ginga 4U0142+61

2. **[40 points]** Figure 1 shows the X-ray spectrum for the Black Hole candidate Ginga 4U0142+61 obtained in 1990 by the Ginga X-ray satellite. Since a black hole is supposed to not emit light, it is believed that the X-rays are produced when gas falling into the black hole frictionally heats up within an accretion disk. Frictional heating of gas in the accretion disk occurs because the velocity of the orbiting gas changes as a function of the radius of the orbit. Gas in a circular orbit at some radius will orbit faster than the gas in a just slightly larger radius orbit and thus it "rubs" against that gas in adjacent orbits, leading to dissipation of kinetic energy into thermal energy. This dissipation of kinetic energy results in the gas in the accretion disk slowing drifting in from an outer radius of several hundred AU to an inner radius of several tens of kilometers. **[HINT:** You can answer parts (c) and (d) without having answered parts (a) and (b).]

- (a) Assume the gas falling into the black hole is in circular orbit around a black hole. Starting with the expression for the velocity of an object in a circular orbit as a function of orbital radius, compute how the velocity changes with radius ( $\frac{dv}{dr}$ ).
- (b) Assume the amount of frictional heating is proportional to  $|\frac{dv}{dr}|$ ,

where in the accretion disk do you expect frictional heating to reach a maximum? *I'm just asking for a verbal description, not an "equation."* Based on this answer, do you believe a large fraction of the accretion disk will be at this high temperature?

- (c) The peak energy of the X-ray spectrum of Ginga 4U0142+61 is about 2.5 keV or about  $4 \times 10^{-16} J$ . What is the corresponding frequency and wavelength for a photon of this energy?
- (d) If we assume the portion of the accretion disk emitting the X-rays observed in this spectrum is dense enough to be in thermal equilibrium and emitting a blackbody spectrum with the peak wavelength corresponding to the peak photon energy, estimate the temperature of this portion of the accretion disk. **(This part of the problem involves material has not yet been covered in the 2009 course, sorry.)**
3. [20 points] Dr. Cabanela's research involves in part determining the motions of stars in the Milky Way. For this work, he uses the spectrometer on Mt. Hopkins that can obtain the spectra of 300 stars simultaneously in visible light. For one of these stars we detect a  $H\alpha$  spectral line ( $\lambda_{rest} = 656.281nm$ ) at a wavelength of  $656.099nm$ . What is the radial velocity of this star? In addition to an actual number, note if the star is approaching or receding and how you know.